Light Source II

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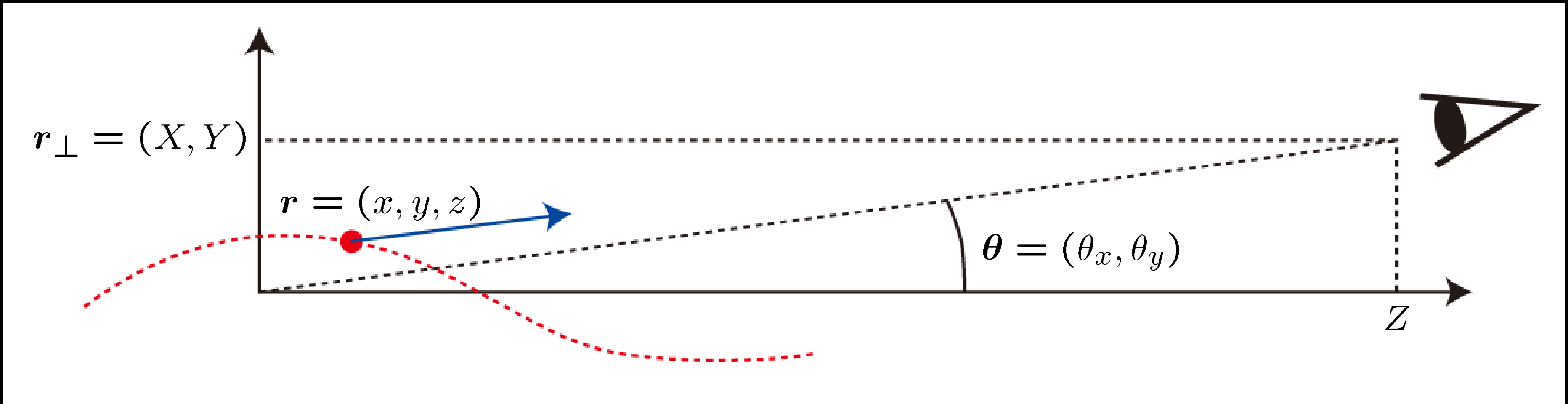
Characteristics of SR (2)

- Electron Trajectory in the Undulator
- Qualitative Description of Undulator Radiation

Coordinate Systems



SR emitted by an electron moving at $\boldsymbol{r} = (x,y,z)$
Observation of SR at $\boldsymbol{R} = (X,Y,Z)$

If the far-field approximation ($|\boldsymbol{r}| \ll Z$) is applicable, the radiation pattern depends only on the observation angle $\boldsymbol{\theta} = (\theta_x, \theta_y)$.

Field Integrals

$$\frac{d\boldsymbol{P}}{dt} = m\gamma\frac{d\boldsymbol{v}}{dt} = -e\boldsymbol{v}\times\boldsymbol{B} \Rightarrow \begin{cases} m\gamma\dot{v}_x = -e(v_yB_z - v_zB_y) \\ m\gamma\dot{v}_y = -e(v_zB_x + v_xB_z) \end{cases}$$

Equation of motion of an electron moving in a magnetic field ***B***

$$\Downarrow\ B_z \equiv 0$$

$$m\gamma\frac{dv_{x,y}}{v_z dt} = m\gamma\frac{dv_{x,y}}{dz} = \pm eB_{y,x}$$

$$\boxed{\begin{aligned} \beta_{x,y} &= \pm\frac{e}{\gamma mc}\int^z B_{y,x}(z')dz' \equiv \pm\frac{e}{\gamma mc}I_{1y,1x}(z) \\ x,y &= \pm\frac{e}{\gamma mc}\int^z\!\!\int^{z'} B_{y,x}(z'')dz'' \equiv \pm\frac{e}{\gamma mc}I_{2y,2x}(z) \end{aligned}}$$

I_1, I_2 : 1st and 2nd field integrals of the undulator

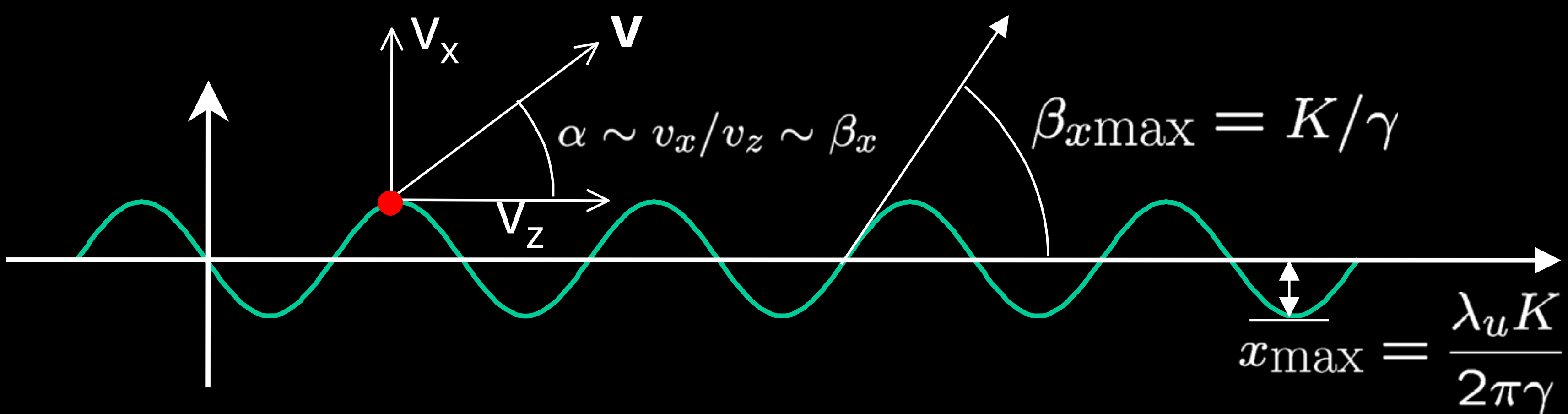
Trajectory in an Ideal Undulator

$$\begin{cases} B_x(z) = 0 \\ B_y(z) \sim B_0 \sin\left(\frac{2\pi z}{\lambda_u}\right) \end{cases} \qquad \begin{cases} \beta_y = 0 \\ \beta_x = \frac{K}{\gamma} \cos\left(\frac{2\pi z}{\lambda_u}\right) \end{cases} \qquad \begin{cases} y = 0 \\ x = \frac{\lambda_u K}{2\pi\gamma} \sin\left(\frac{2\pi z}{\lambda_u}\right) \end{cases}$$

magnetic field → velocity → position

$$K = \frac{eB_0\lambda_u}{2\pi mc} = 93.37 B_0(\mathrm{T})\lambda_u(\mathrm{cm})$$

K value, Deflection parameter



E_e=8GeV,K=1,λ_u=5cm : β_{xmax}=64μrad,x_{max}=0.5μm

Effects due to the Undulator Field

transverse velocity

$$\beta_x = \frac{K}{\gamma}\cos\left(\frac{2\pi z}{\lambda_u}\right)$$

longitudinal velocity

$$\beta_z = \sqrt{\beta^2 - \beta_x^2}$$

total velocity

$$= \underbrace{\boxed{1 - \frac{1}{2\gamma^2}} - \frac{K^2}{4\gamma^2}}_{\bar{\beta}_z} - \underbrace{\frac{K^2}{4\gamma^2}\cos\left(\frac{4\pi z}{\lambda_u}\right)}_{\text{oscillating term}}$$

$\bar{\beta}_z$:average velocity

oscillating term

Undulator field induces:

- transverse(x) oscillation
- longitudinal (z) oscillation
- effective deceleration($\Delta\beta_z = K^2/4\gamma^2$)

Electron Motion: Two Forms

$$\beta_x = \frac{K}{\gamma}\cos\left(\frac{2\pi z}{\lambda_u}\right)$$

- Horizontal oscillation with a period of λ_u
- Major contribution to radiation

$$\beta_z = \bar{\beta}_z - \frac{K^2}{4\gamma^2}\cos\left(\frac{4\pi z}{\lambda_u}\right)$$

- Longitudinal oscillation with a period of $\lambda_u/2$
- Amplitude $1/\gamma$ times lower than β_x.
- Minor contribution, but source of vertical polarization observed vertically off-axis.

General Form of Time Squeezing

$$\frac{d\tau}{dt} = 1 - \boldsymbol{\beta} \cdot \boldsymbol{n}$$

$$\boxed{\begin{aligned} \beta_z &= \sqrt{\beta^2 - \beta_x^2 - \beta_y^2} \\ &\sim 1 - (\gamma^{-2} + \beta_x^2 + \beta_y^2)/2 \\ n_z &\sim 1 - (\theta_x^2 + \theta_y^2)/2 \end{aligned}}$$

$$= \frac{1}{2\gamma^2} + (\theta_x - \beta_x)^2 + (\theta_y - \beta_y)^2$$

Time squeezing takes place most significantly when the direction of the electron motion coincides with that of observation ($\beta = \theta$).

Characteristics of SR (2)

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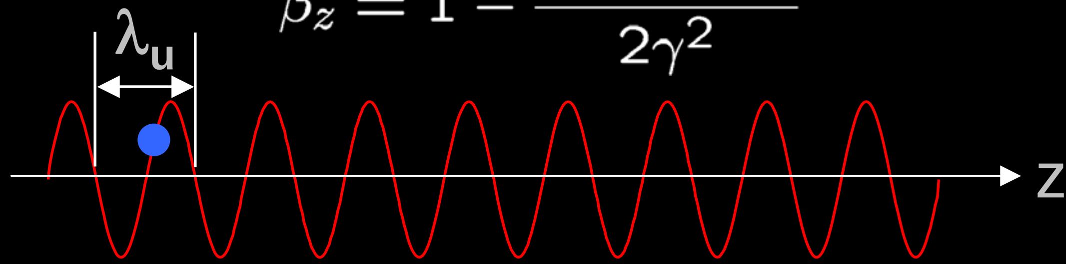
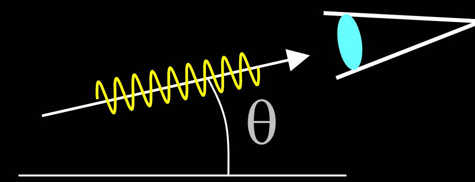
Fundamental Wavelength

$$\bar{\beta}_z = 1 - \frac{1+K^2/2}{2\gamma^2}$$





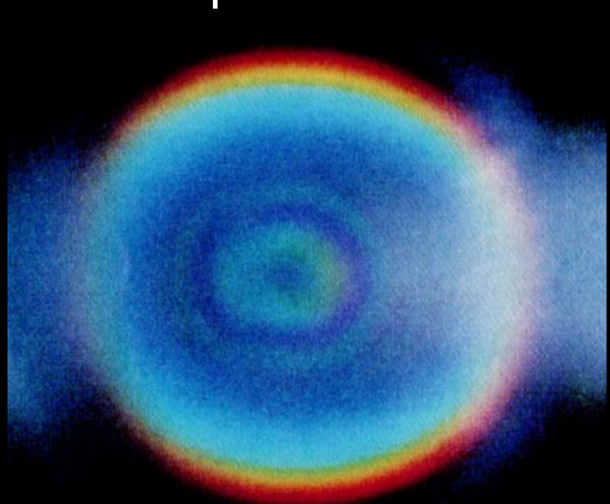
$$T = \lambda_u/v_z = \lambda_u/c$$

period of electron motion
= period of emitted light

time squeezing

$$T' = T(1-\bar{\beta}_z\cos\theta)$$

period of observed light



H. Kitamura et al.,
J. Appl. Phys. 21 (1982) 1728

Fundamental Wavelength λ_1

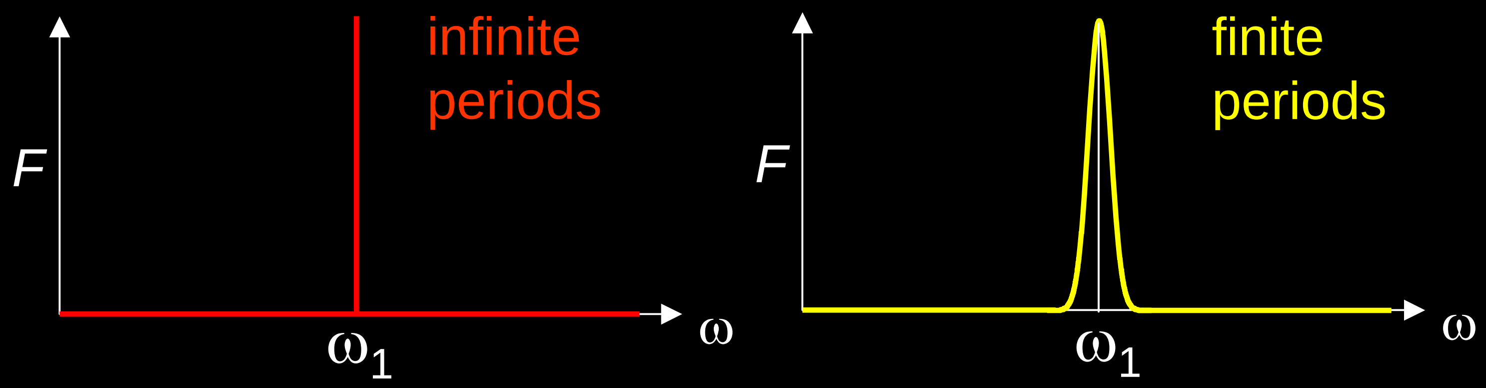
$$\begin{aligned}\lambda_1 &= \lambda_u(1-\bar{\beta}_z\cos\theta)\\ &= \frac{\lambda_u}{2\gamma^2}(1+\gamma^2\theta^2+K^2/2)\\ \omega_1 &= 2\pi c/\lambda_1\end{aligned}$$

UR with Infinite Periods

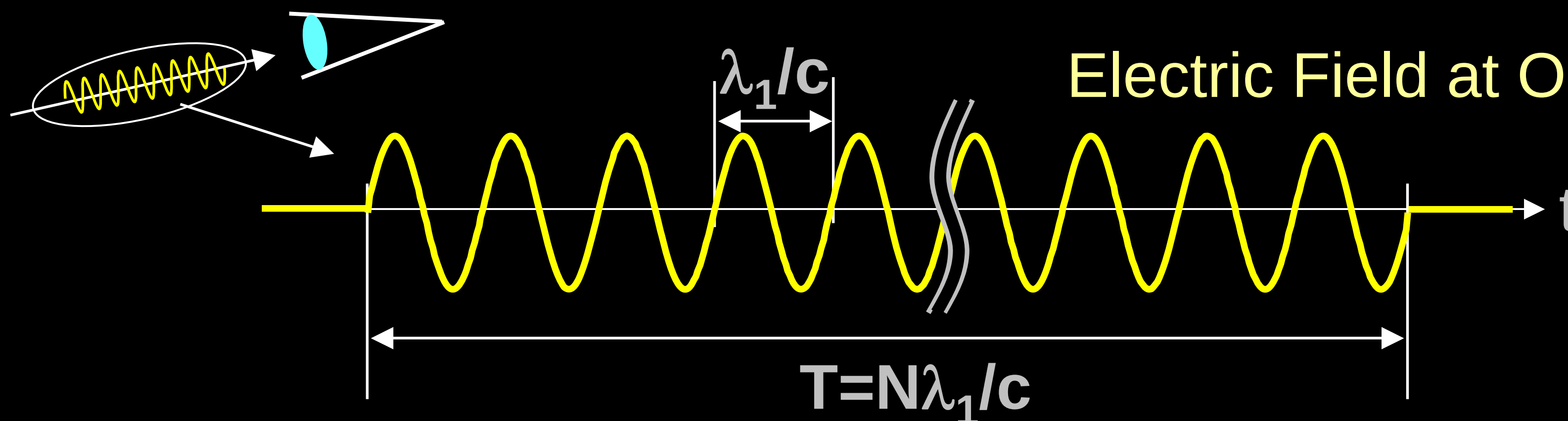
- If the undulator length is infinite, the pulse duration is infinitely long, and thus the radiation is completely monochromatic with line spectrum.

$$\frac{d^2F}{dx'dy'} \propto \delta(\omega-\omega_1) = \delta\left(\omega - \frac{4\pi c\gamma^2/\lambda_u}{1+K^2/2+\gamma^2\theta^2}\right)$$

- In practice, the undulator length is finite, so the line spectrum is broadened.



Effects due to Finite Periods



$$E(t) = \begin{cases} E_0 \sin \omega_1 t & ; -T/2 \le t \le T/2 \\ 0 & ; t < -T/2, T/2 < t \end{cases}, \quad \omega_1 = 2\pi c/\lambda_1$$

Fourier Transform

$$\frac{d^2F}{dx'dy'} \propto |\tilde{E}(\omega)|^2 \propto \mathrm{sinc}^2 \left[\pi N \frac{\omega - \omega_1(\theta)}{\omega_1(\theta)} \right]$$

Square of “sinc” function dominates the UR

Brief Note on UR Formulae

- In the previous derivations of UR spectral function, no knowledge on electro-dynamics is required.
- In practice, E_0 is a complicated function of θ and K, and needs to be calculated by Fourier transforming the electric field derived from the Lienard-Wiecherd potential.
- However, the simple derivation gives us a clear understanding on UR properties.

Energy and Angular Profile of UR

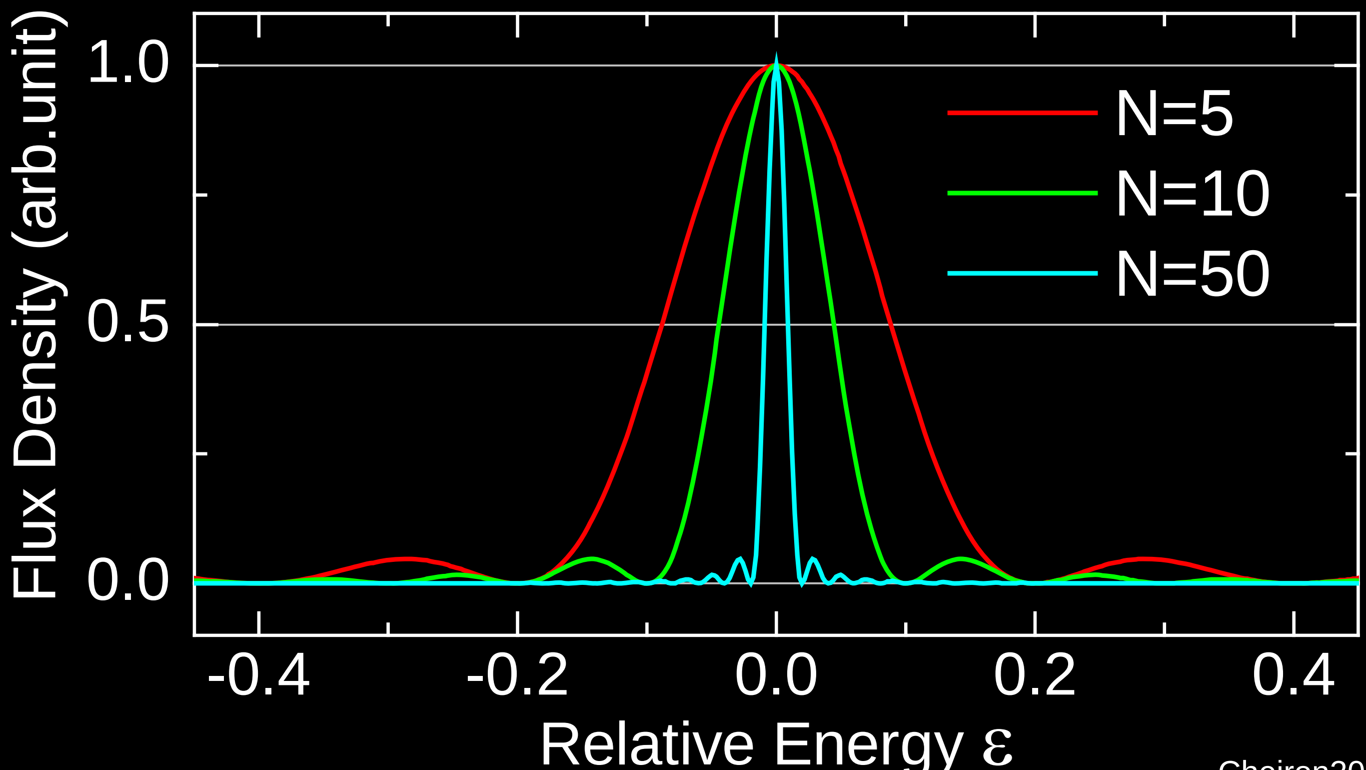
$$\frac{d^2F(\omega,\theta)}{d\Omega d\omega/\omega} = F_0 \mathrm{sinc}^2\left[\pi N \frac{\omega - \omega_1(\theta)}{\omega_1(\theta)}\right]$$

$$= \begin{cases} \boxed{\begin{array}{l} \text{Energy Profile at } \theta = 0 \\ F_0 \mathrm{sinc}^2(N\pi\varepsilon) \\ \quad ;\varepsilon = [\omega - \omega_1(0)]/\omega_1(0) \end{array}} \\ \boxed{\begin{array}{l} \text{Angular Profile at } \omega = \alpha\omega_1(0) \\ F_0 \mathrm{sinc}^2[N\pi(\alpha\Theta^2 + \alpha - 1)] \\ \quad ;\Theta = \gamma\theta/\sqrt{1 + K^2/2} \end{array}} \end{cases}$$

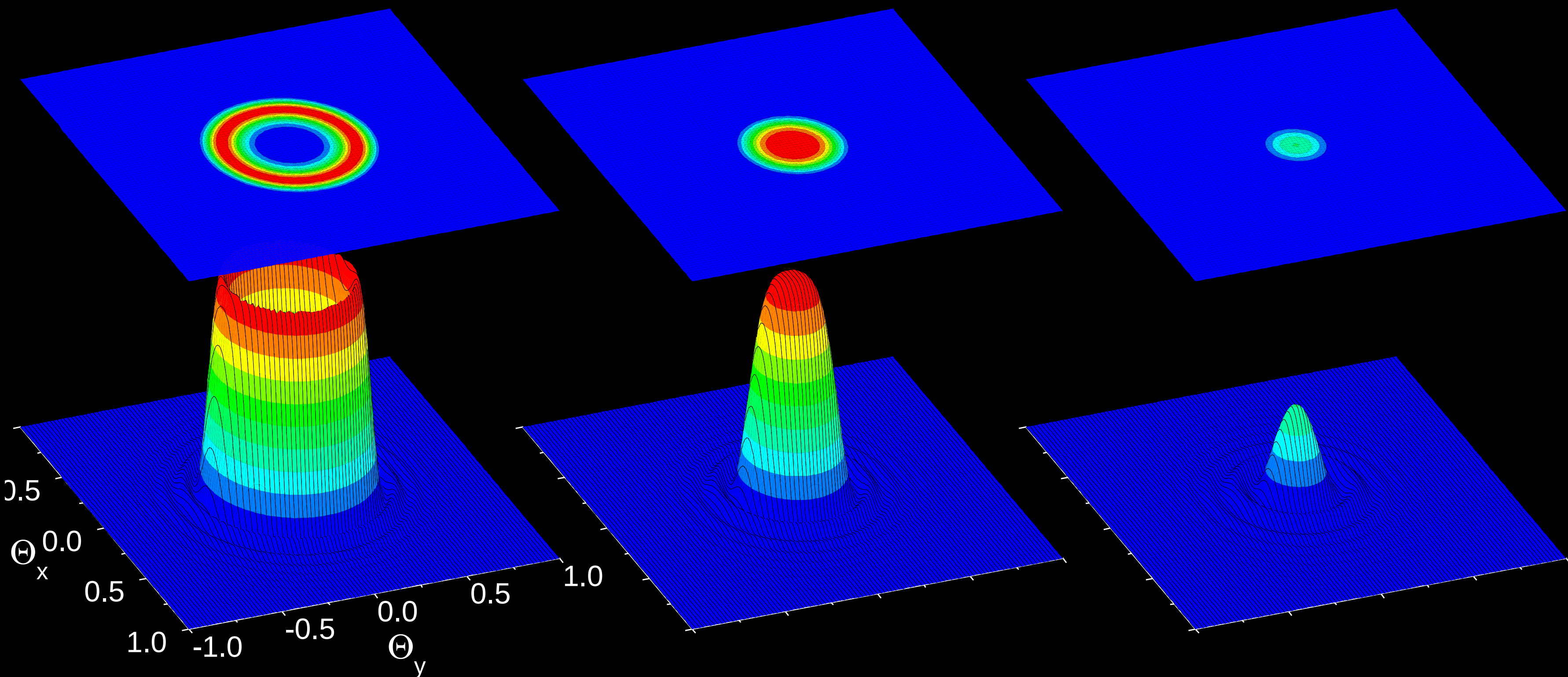
Energy Profile: Example

$$\frac{d^2F}{dx'dy'} = F_0 \mathrm{sinc}^2(N\pi\varepsilon); \quad \mathrm{sinc}^2(2.783) \sim 1/2$$

$$\Rightarrow \left.\frac{\Delta\omega}{\omega_1(0)}\right|_{FWHM} \sim \frac{0.8858}{N}$$



Angular Profile: Example

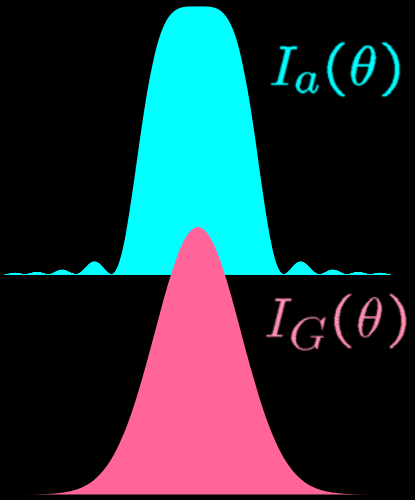


$\omega=0.9\omega_1(0)$	$\omega=\omega_1(0)$	$\omega=1.05\omega_1(0)$
lower energy	fundamental energy	higher energy

Angular Divergence and Beam Size

Angular Profile at $\omega=\omega_1(0)$

$$I_a(\theta) = F_0 \mathrm{sinc}^2\left[\frac{\pi N(\gamma\theta)^2}{1+K^2/2}\right]$$



approximation

Gaussian Profile with $\sigma_{r'}$

$$I_G(\theta) = F_0 \exp(-\theta^2/2\sigma_{r'}^2)$$

$$\sigma_{r'} = \sqrt{\frac{1+K^2/2}{4N\gamma^2}} = \sqrt{\frac{\lambda_1}{2L}}$$

Angular Divergence of UR ($L=N\lambda_u$)

Diffraction Limit (UR is Spatially Coherent)

$$\sigma_r = \frac{\lambda_1}{4\pi\sigma_{r'}} = \frac{\sqrt{\lambda_1 L}}{4\pi}$$

Beam Size of UR

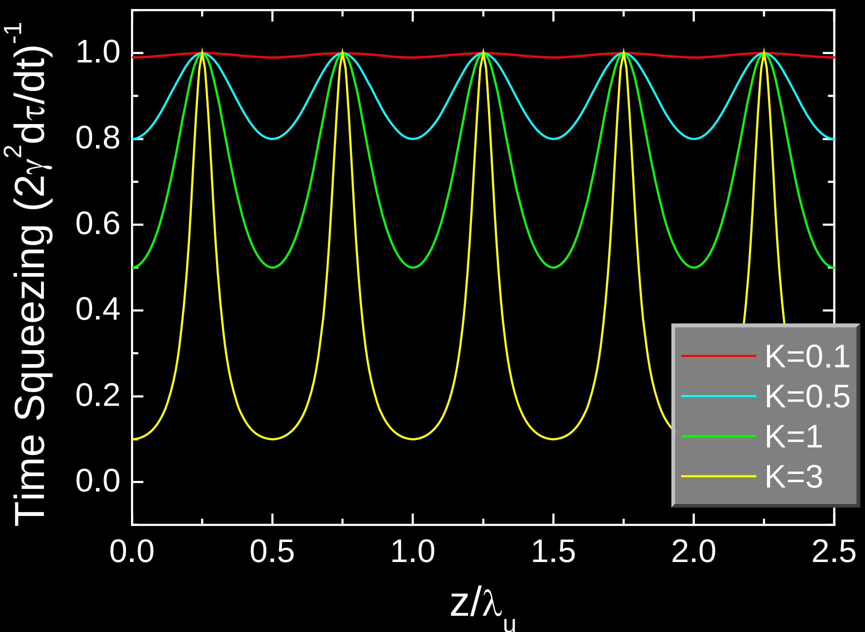
Higher Harmonics

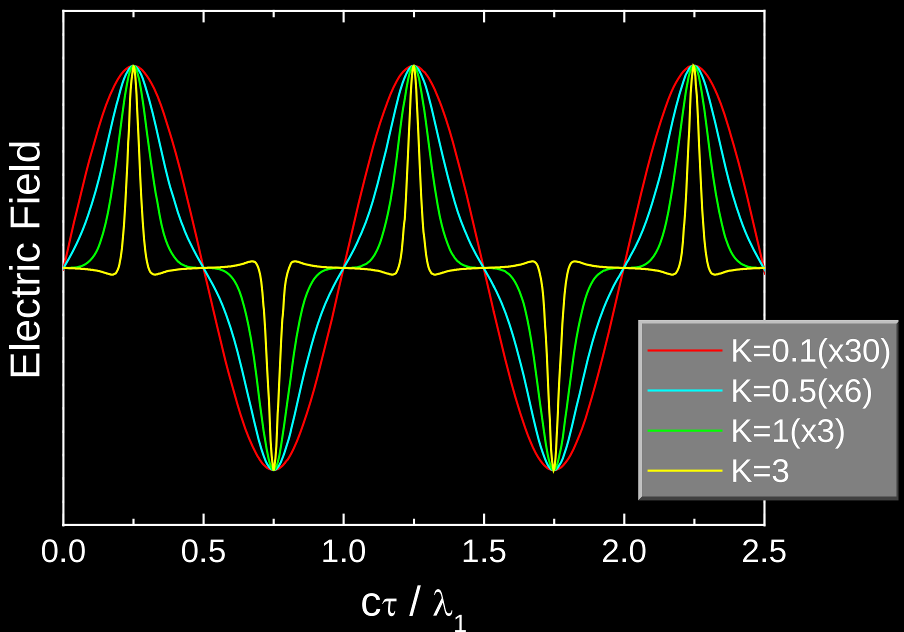
- In addition to the fundamental radiation at ω_1, higher-energy radiation at $n\omega_1$, called higher harmonics, is observed. The integer *n* is referred to as a harmonic number.
- This is a consequence of the fact that the time-squeezing factor depends on the longitudinal electron position and thus the electric field in the time domain is distorted.

Interpretation of Higher Harmonics

$$\frac{d\tau}{dt} = 1 - \boldsymbol{\beta}\cdot\boldsymbol{n} = \frac{1}{2\gamma^2}\left[1 + K^2\cos^2(2\pi z/\lambda_u)\right]$$

on-axis observation:n=(0,0,1)

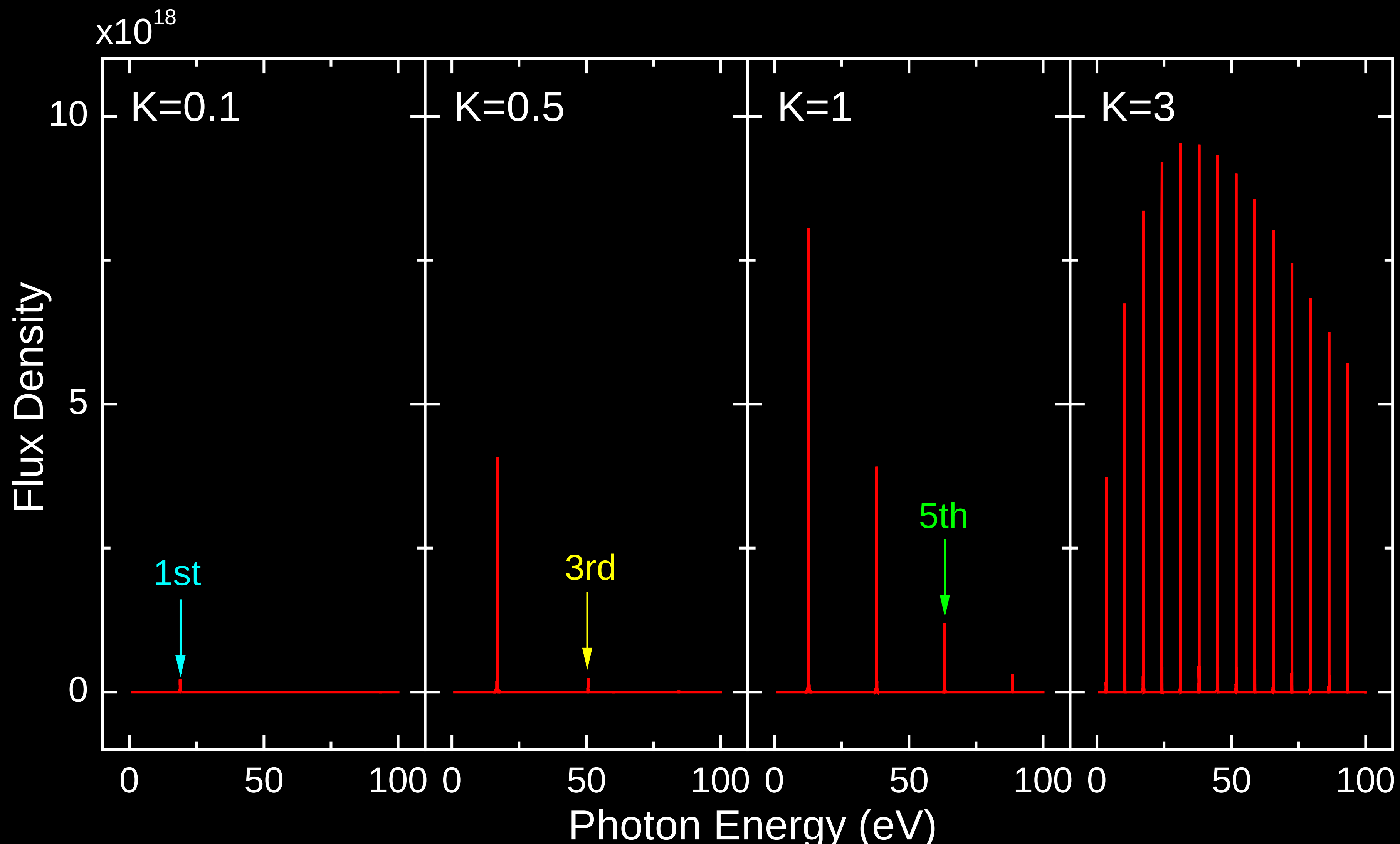




Large K value brings a modulation in the time squeezing factor

Distortions of the electric field takes place due to the nonuniform time squeezing. Due to symmetry, even harmonics do not appear.

Examples of Higher Harmonics



Optical Properties of Higher Harmonics

For the n-th harmonic radiation,

$$\frac{d^2F}{dx'dy'} = F_0 \mathrm{sinc}^2\left[\pi n N \frac{\omega - n\omega_1(\theta)}{n\omega_1(\theta)}\right]$$

$$\left.\frac{\Delta\omega}{n\omega_1(0)}\right|_{FWHM} \sim \frac{0.8858}{nN}$$ band width

$$\sigma_{r'n} = \sqrt{\frac{1+K^2/2}{4nN\gamma^2}} = \sqrt{\frac{\lambda_1/n}{2L}}$$ angular divergence

$$\sigma_{rn} = \frac{\lambda_1/n}{4\pi\sigma_{r'n}} = \frac{\sqrt{L\lambda_1/n}}{4\pi}$$ beam size

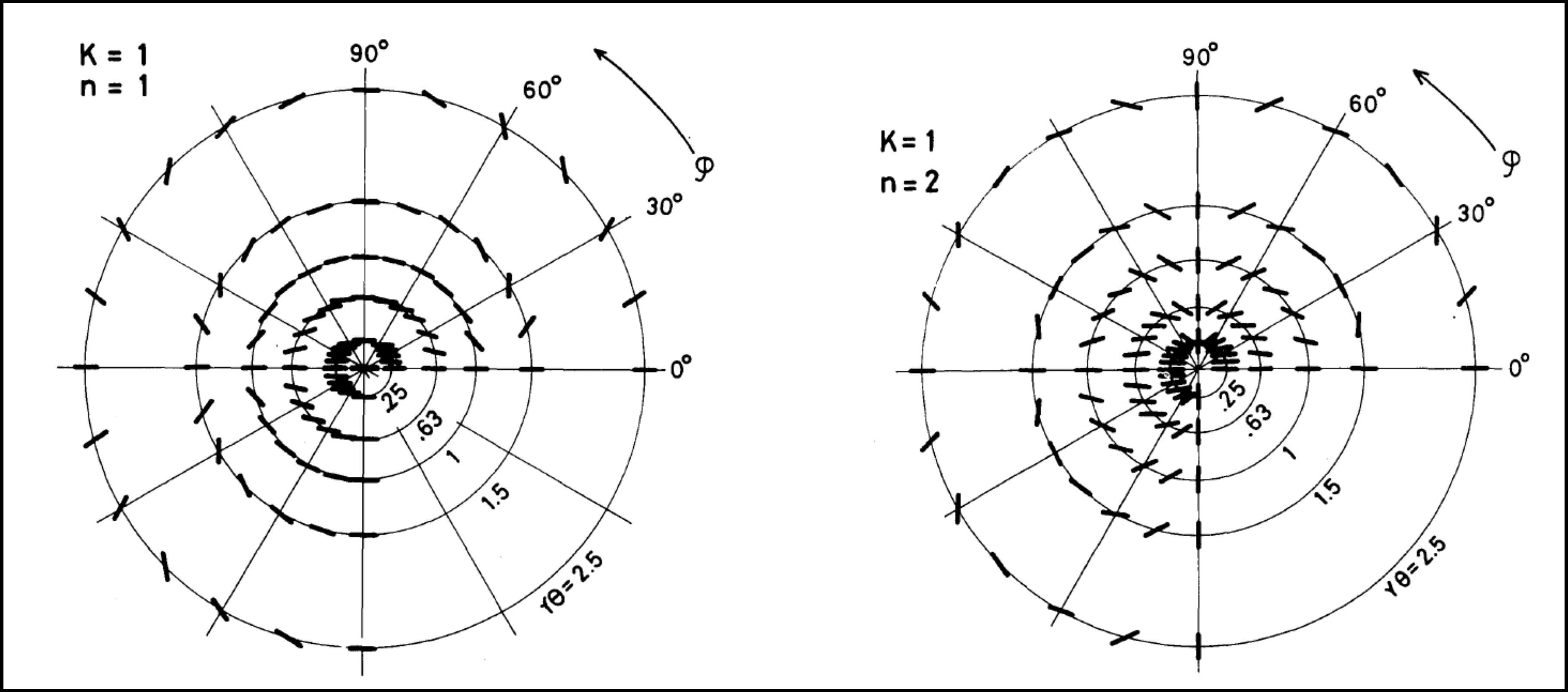
Polarization

- As in the wigglers, no circular polarized radiation (CPR) is observed due to cancellation of CPR components.
- The direction of the linear polarization observed off axis is tilted due to the longitudinal oscillation of electron motion.

Polarization:Examples

Examples of the direction of linear polarization for various observation angles.

H. Kitamura, JJAP 19 (1980) L185

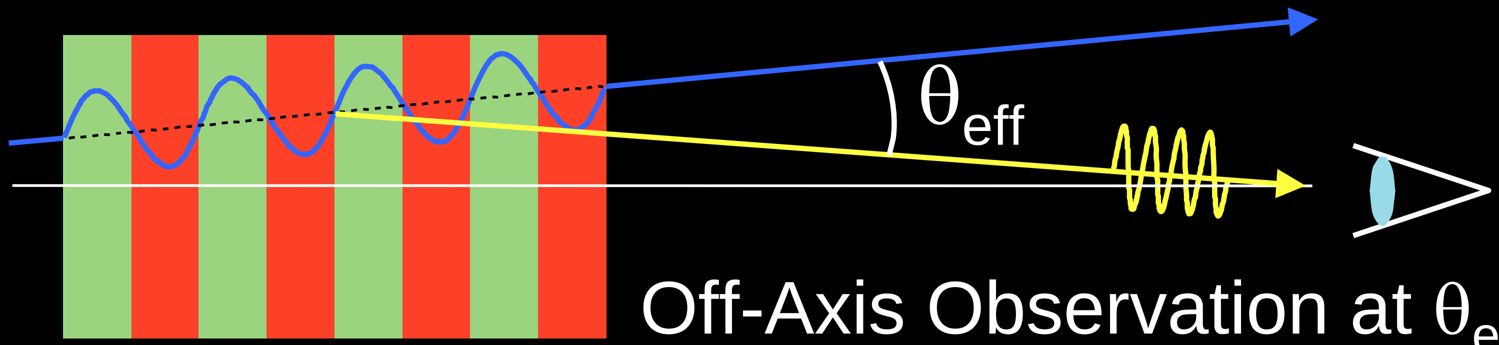


Practical Knowledge on SR

Effects due to Finite Emittance (1)

- Effects due to Finite Emittance of the Electron Beam
  - Injection to the undulator with angular and positional offset

SR with Position and Slope Offset



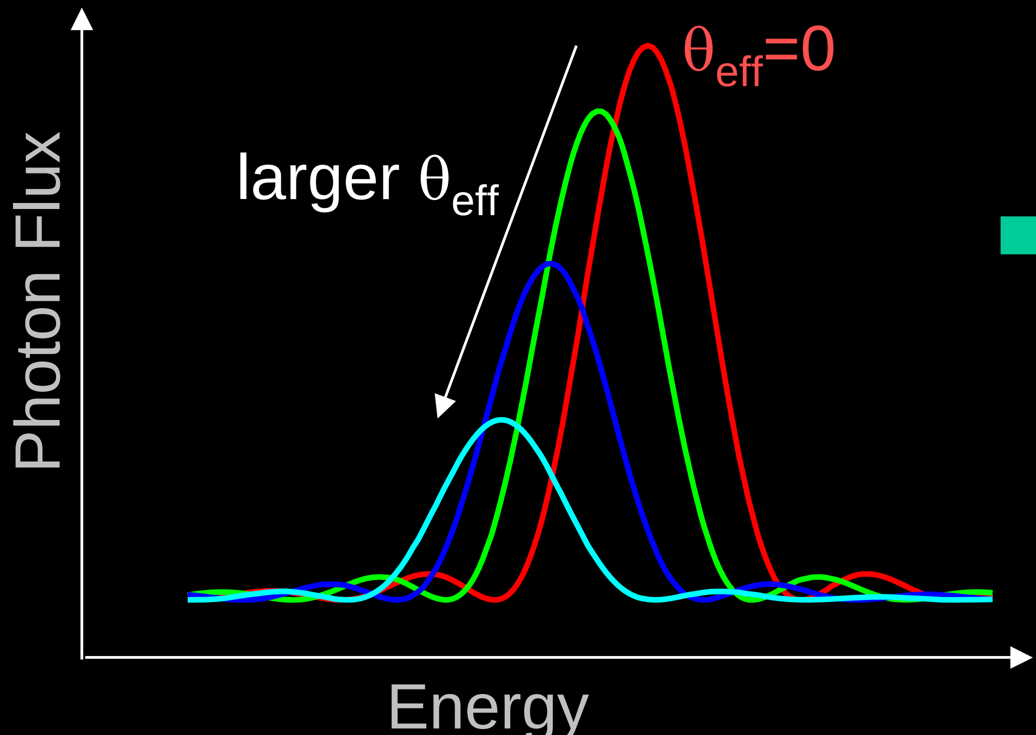
Off-Axis Observation at θ_{eff}

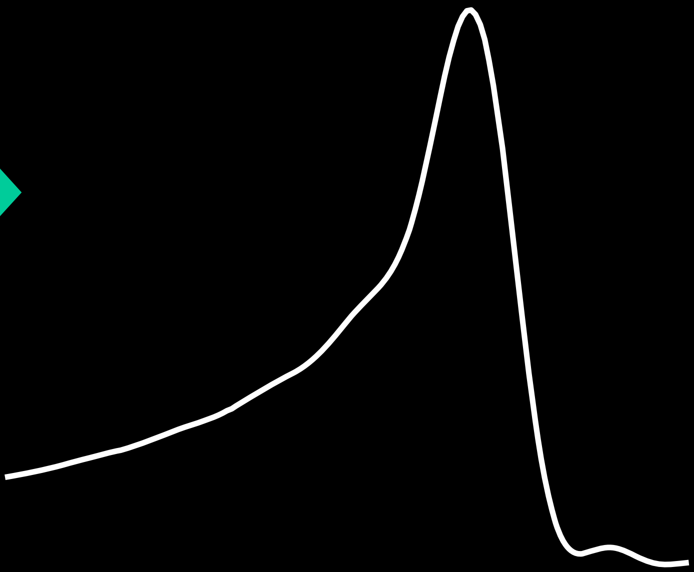
Effects due to Finite Emittance (2)

Off-axis observation at θ_{eff}

→ Peak shift to lower energy

$$\omega_1(\theta) = \frac{4\pi c \gamma^2/\lambda_u}{1 + \boxed{\gamma^2\theta^2} + K^2/2}$$



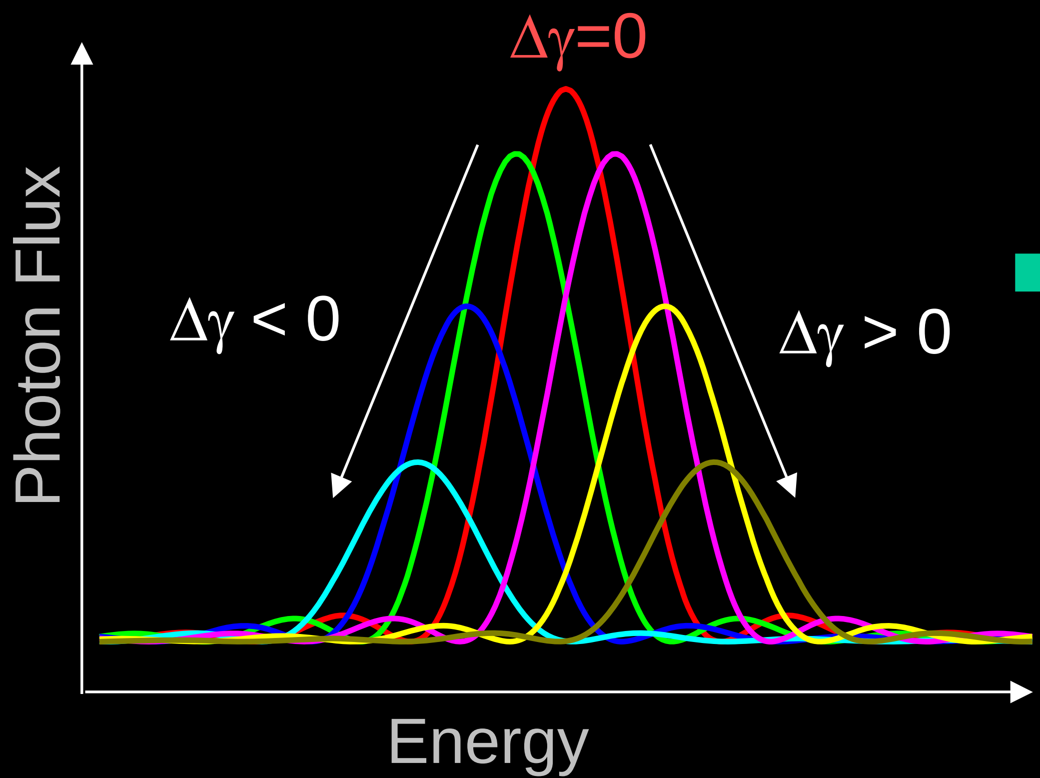


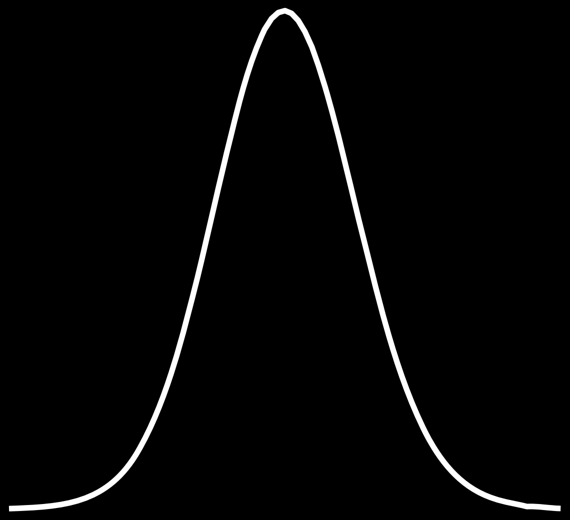
Effects due to the Energy Spread

Electron with an offset of $\Delta\gamma$

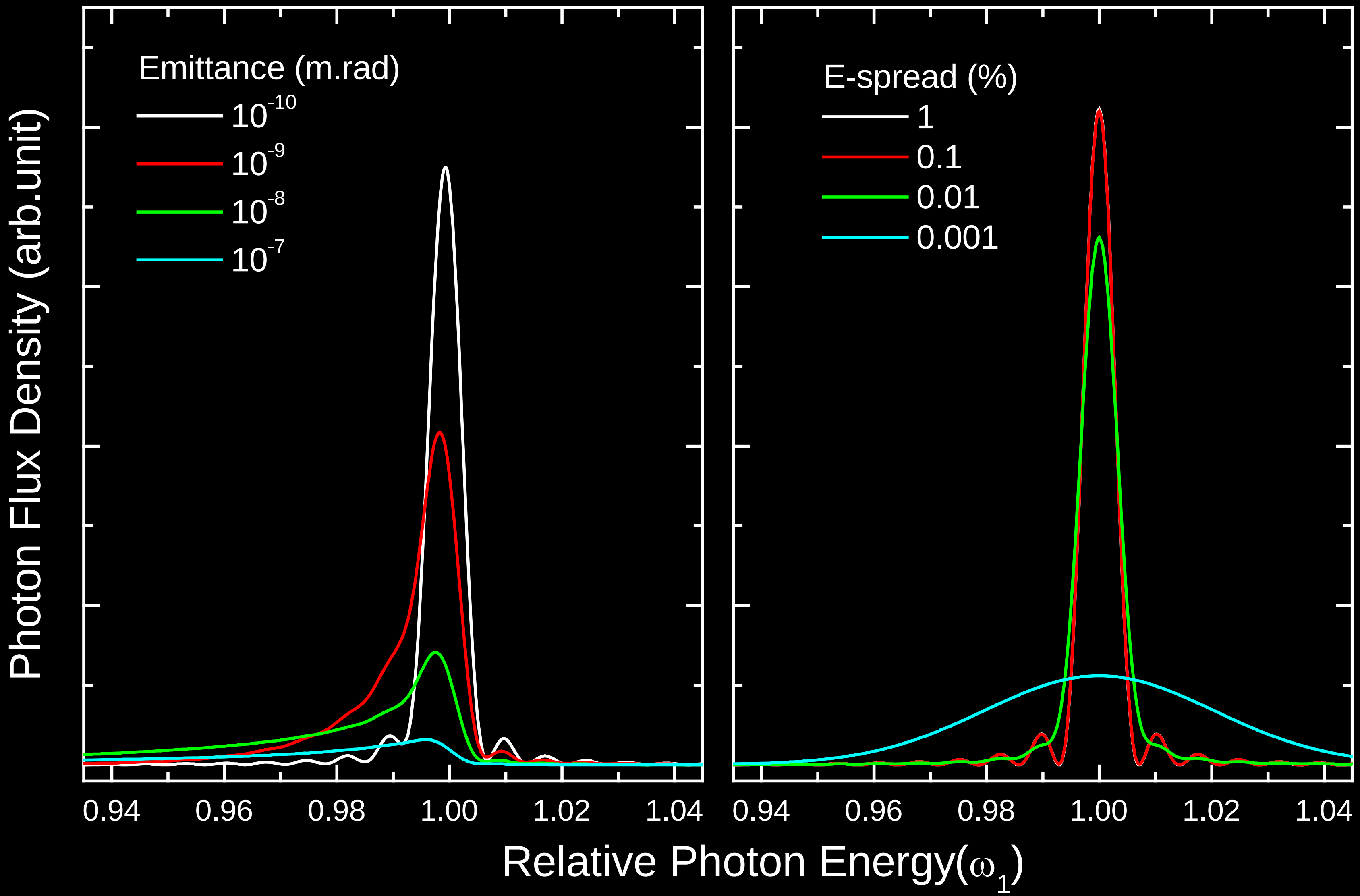
→ Energy shift of ω_1

$$\omega_1(\gamma) = \frac{4\pi c\boxed{\gamma^2}/\lambda_u}{1+K^2/2}$$





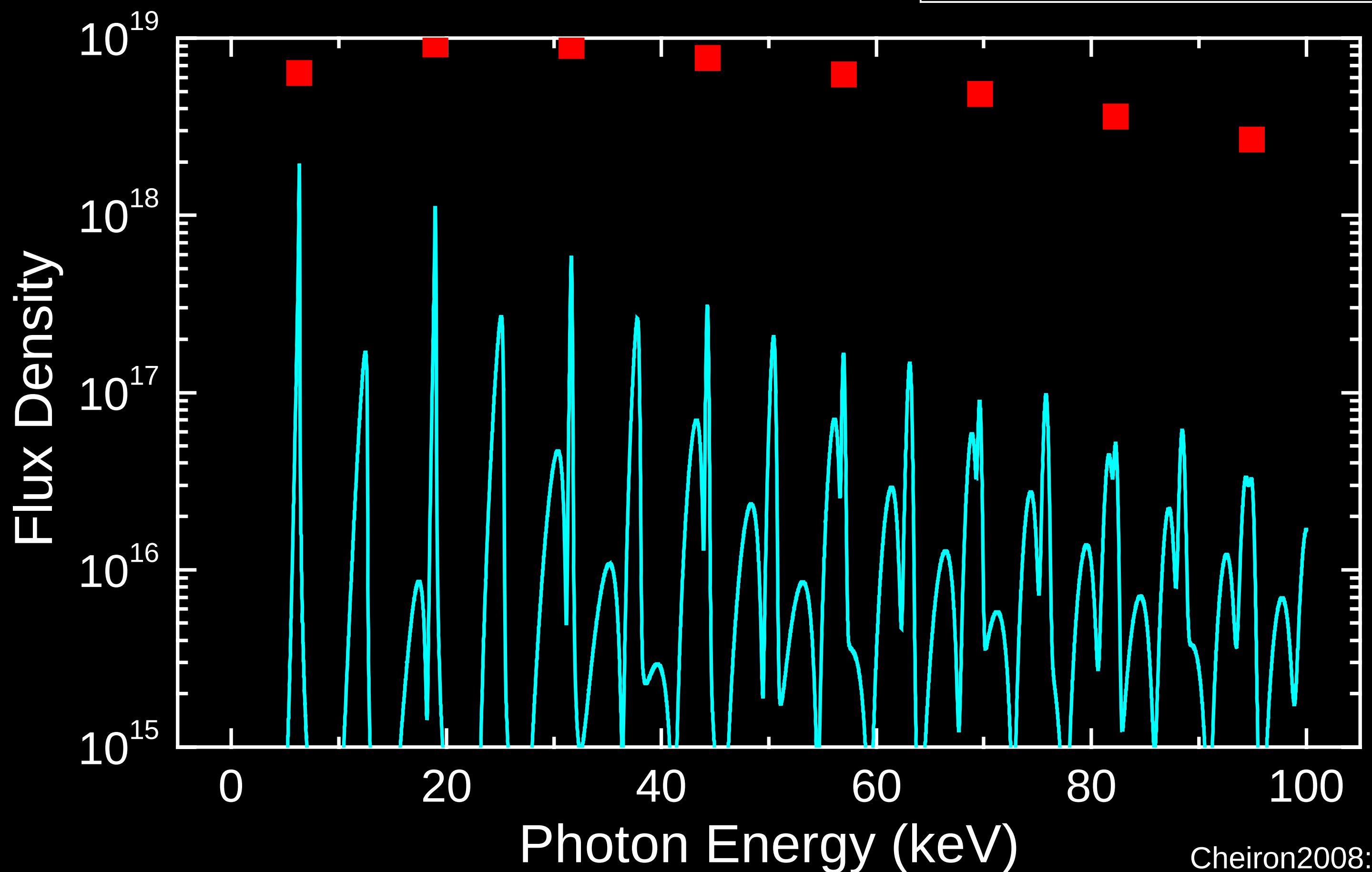
Examples


Emittance (m.rad)
10^{-10}
10^{-9}
10^{-8}
10^{-7}
E-spread (%)
1
0.1
0.01
0.001
Photon Flux Density (arb.unit)
0.94
0.96
0.98
1.00
1.02
1.04
Relative Photon Energy(ω_1)

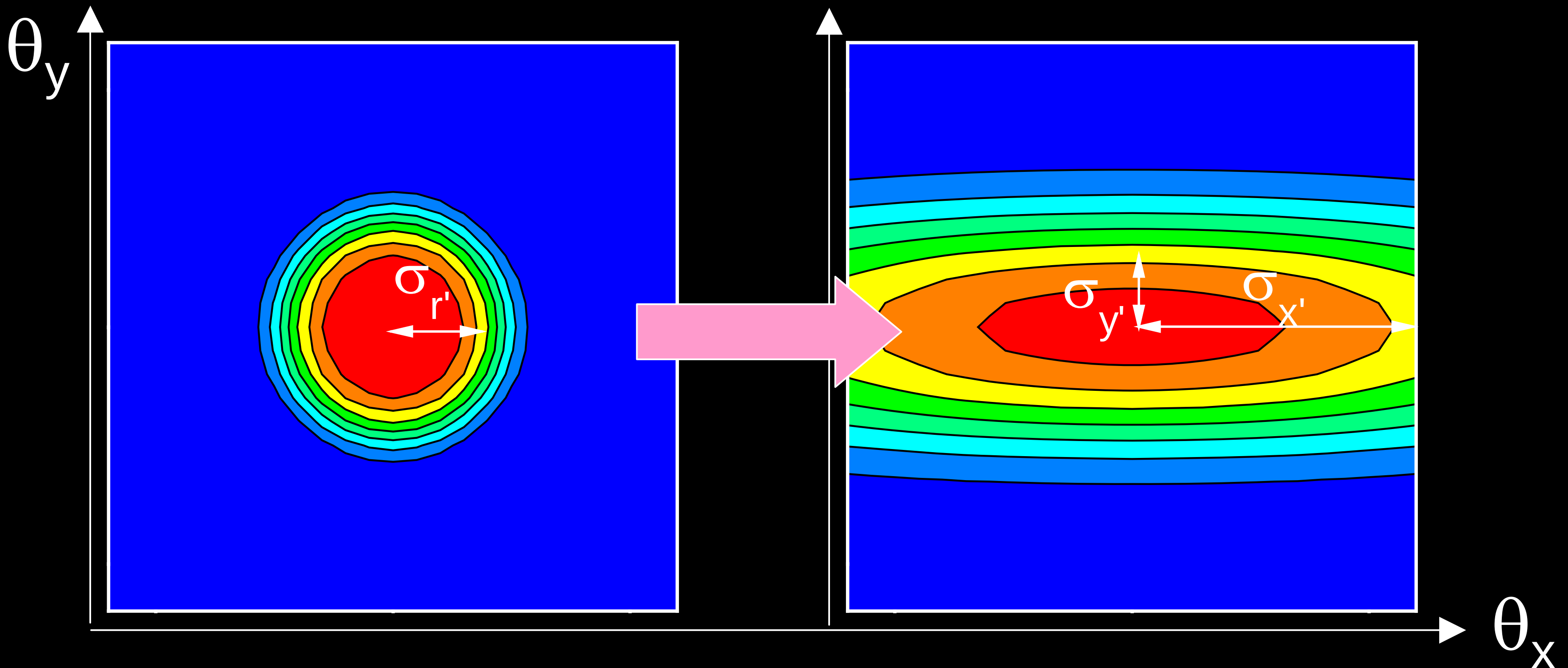
Effects on the Higher Harmonics

Optical Emittance of UR: $\lambda/4\pi$
Bandwidth of UR: *~1/nN*

Effects due to the e^- beam are larger for higher harmonics



Effective Beam Size and Divergence



Under Gaussian approximation

$$\sigma_{x',y'} = \sqrt{\sigma_{r'}^2 + \sigma_{ex',ey'}^2}, \quad \sigma_{x,y} = \sqrt{\sigma_r^2 + \sigma_{ex,ey}^2}$$

*effective beam size　　　*effective divergence

*Note: Valid only near the resonance energy ($n\omega_1$).

Effective Flux Density and Brilliance

Simple scheme to estimate the on-axis flux density and brilliance.

Total Flux

$$F = \boxed{\left.\frac{d^2F}{dx'dy'}\right|_0} \times 2\pi\sigma_{r'}^2$$

on-axis flux density with zero-emittance beam

Effective Flux Density

$$\left.\frac{d^2F}{dx'dy'}\right|_e = \frac{F}{2\pi\sigma_{x'}\sigma_{y'}} = \left.\frac{d^2F}{dx'dy'}\right|_0 \frac{\sigma_{r'}^2}{\sigma_{x'}\sigma_{y'}}$$

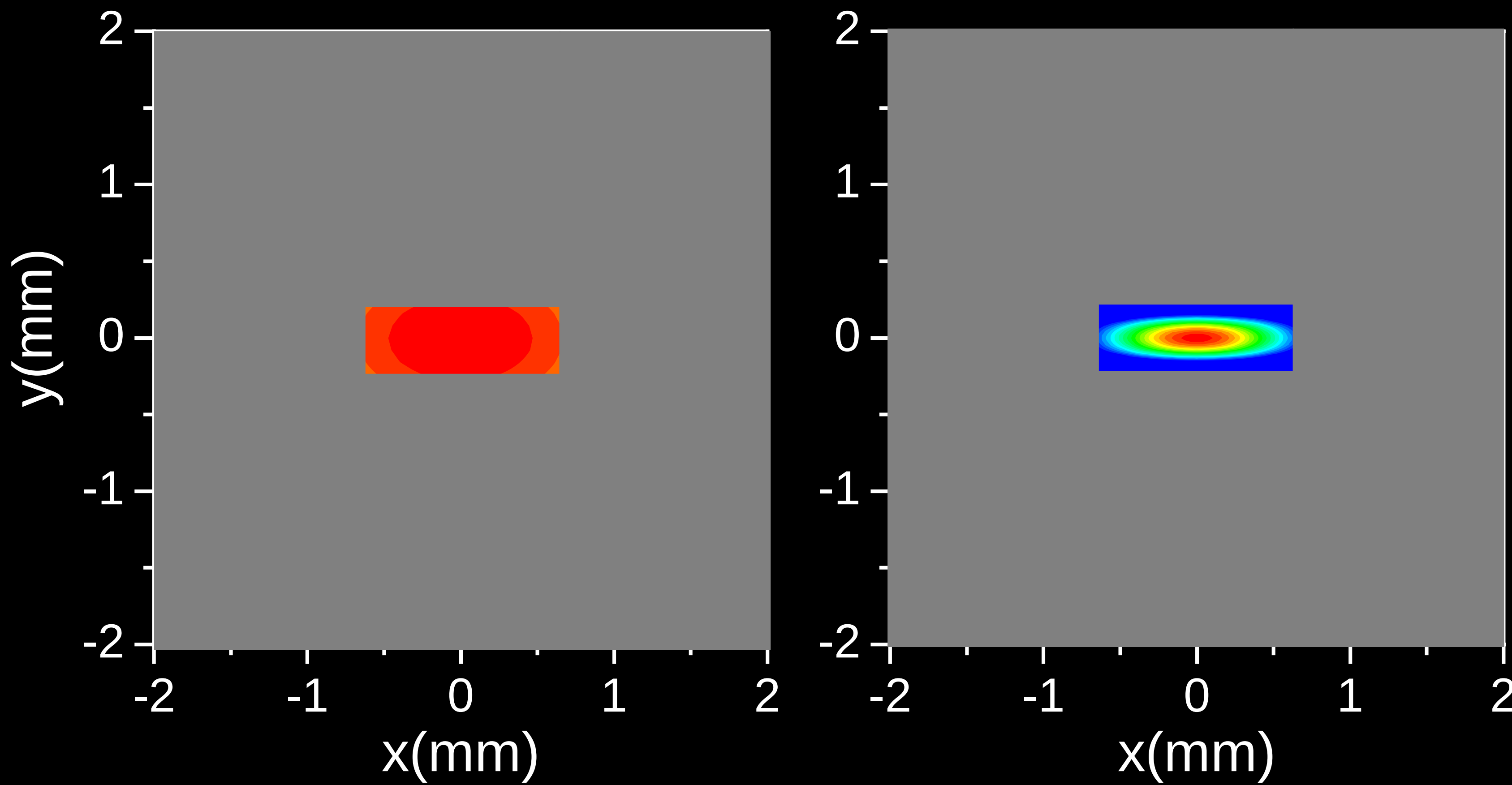
Effective Brilliance

$$B_e = \frac{F}{4\pi^2\sigma_x\sigma_y\sigma_{x'}\sigma_{y'}} = \left.\frac{d^2F}{dx'dy'}\right|_0 \frac{\sigma_{r'}^2}{2\pi\sigma_x\sigma_{x'}\sigma_y\sigma_{y'}}$$

Heat Load on Optical Elements

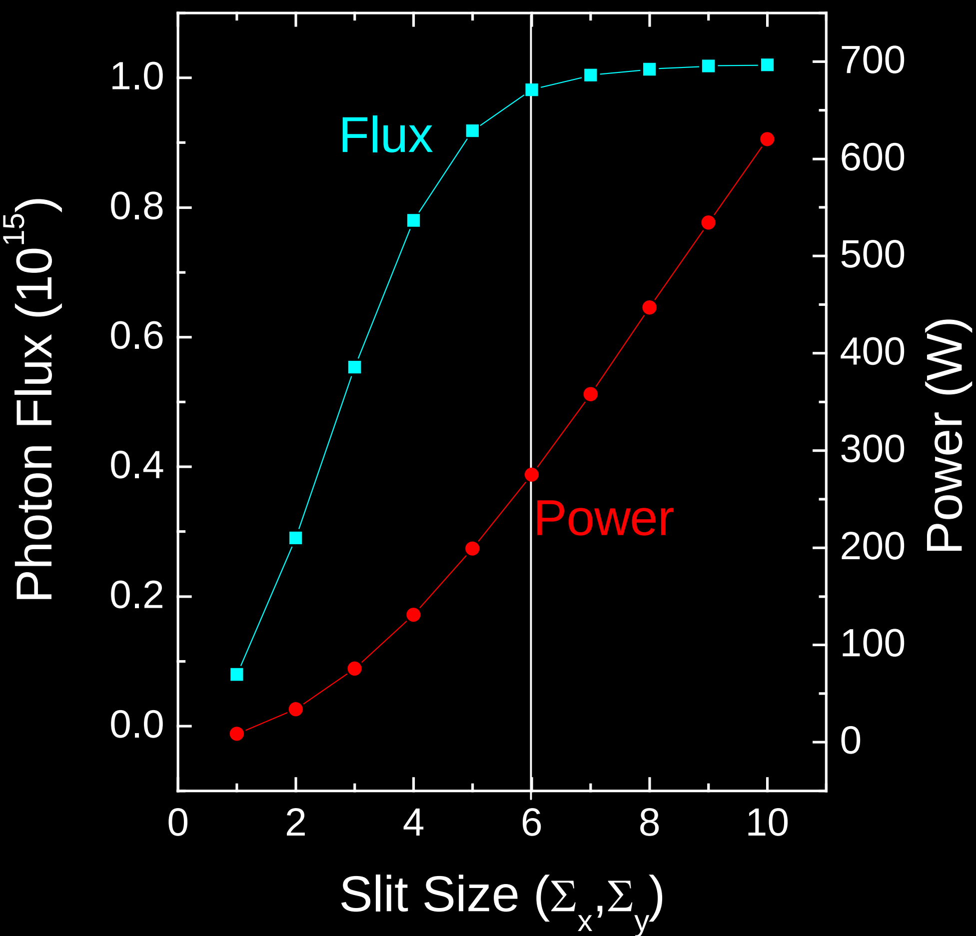
- SR emitted from the light source is processed by several optical elements before irradiation to the sample, such as the focusing mirror, monochromator.
- These elements can be easily damaged by the heat load brought by the SR.
- It is thus important to reduce the heat load as much as possible without sacrificing the flux, which is actually done by the XY slit at the front-end section.

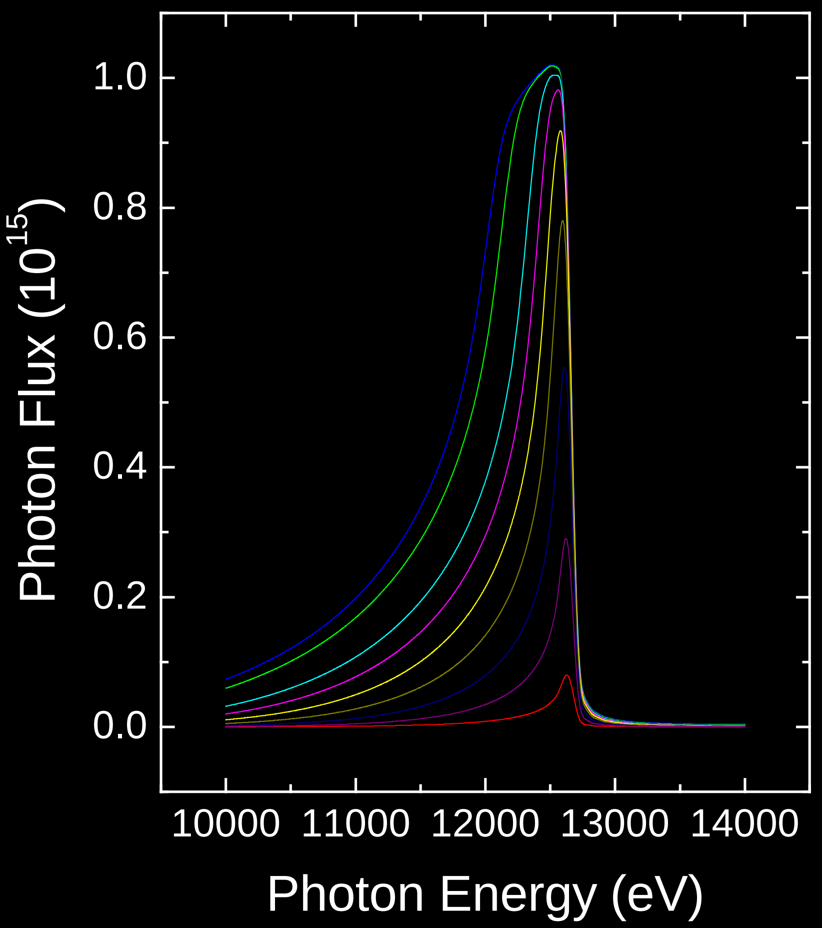
Spatial Profile of Power and Flux



The power profile is much broader than the flux. Extraction of SR with an appropriate slit significantly reduces the heat load.

Optimum Slit Size?

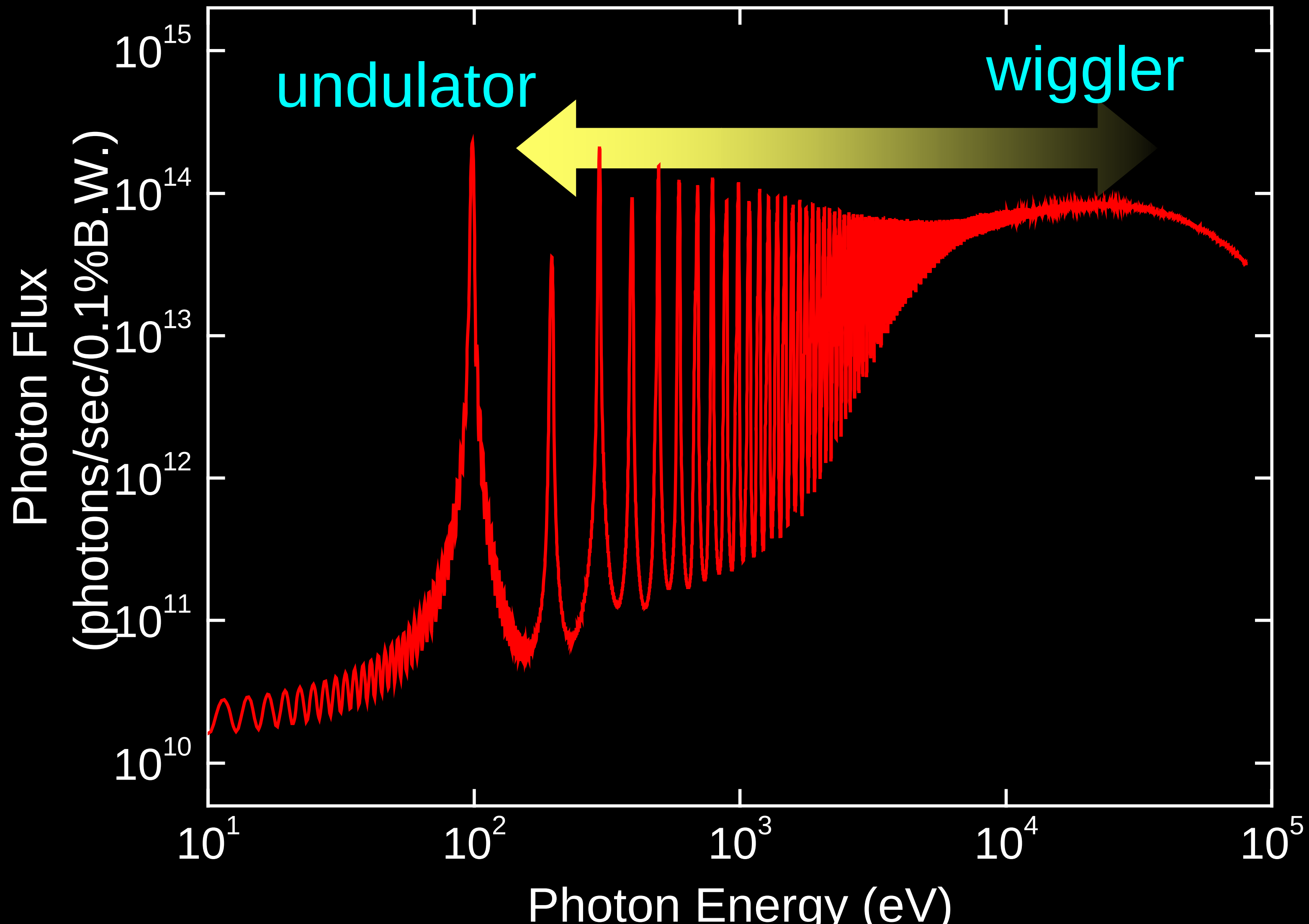




Wiggler? Undulator? (1)

- Wigglers are identical to undulator from the point of view of magnetic circuit.
- It is generally said that the K value distinguishes between the two, however, this is not exactly correct.
- What we should take care is the region of photon energy to be utilized for application.

Wiggler? Undulator? (2)



Other Topics Not Addressed

- Quantitative descriptions of SR
- Light sources for circular polarization and schemes for fast helicity switching
  – helical undulator & elliptic wiggler
  – chicanes&choppers, kicker magnets
- Effects on the electron beam
  – natural focusing
  – beam-axis fluctuation due to COD variation
- R&Ds toward shorter magnetic period
  – superconducting undulators
  – cryogenic permanent magnet undulators
- Coherent SR for intense THz light
- Undulators for SASE-based X-ray FEL