

Light Source I

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Light Source I



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- Fundamentals of Light and SR
- Overview of SR Light Source
- Characteristics of SR (1)

Light Source II



- Characteristics of SR (2)
- Practical and Useful Knowledge

Introduction

SR Facility and Light Source

- SR: Definition
 - Electromagnetic wave emitted by a charged particle deflected by a magnetic force
- SR Facility
 - Accelerators to generate a high-energy electron beam
 - **Magnetic devices (SR light source) to generate intense SR**
 - Optical elements (monochromators, mirrors,..)
 - Experimental stations

SR as a Probe for Research

- SR has a lot of advantages over other conventional light sources
 - Highly collimated (laser-like)
 - Wavelength tunability
 - Polarization
 -
- However, the total radiation power does not differ significantly.



Comprehensive understanding of SR (and light source) is required for efficient experiments.

Topics in This Lecture (1)

- Fundamentals of Light and SR
 - General description
 - Physical quantity of light
 - Uncertainty of light: Fourier and diffraction limits
 - SR: Light from a moving electron
- Overview of SR Light Source
 - Types of light sources
 - Magnet configuration
- Characteristics of SR (1)
 - Radiation from bending magnets
 - Radiation from wigglers

Topics in This Lecture (2)

- Characteristics of SR (2)
 - Radiation from undulators
 - Electron motion in the undulator
 - Fundamental energy
 - Spectral function
 - Spatial profile of radiation
 - Higher harmonics
- Practical and Useful Knowledge on SR
 - Finite emittance and energy spread
 - Heat load and photon flux
 - Definition of undulators and wigglers

Fundamentals of Light and SR

- General Description
- Physical Quantity of Light
- Uncertainty of Light
- SR: Light from a Moving Electron

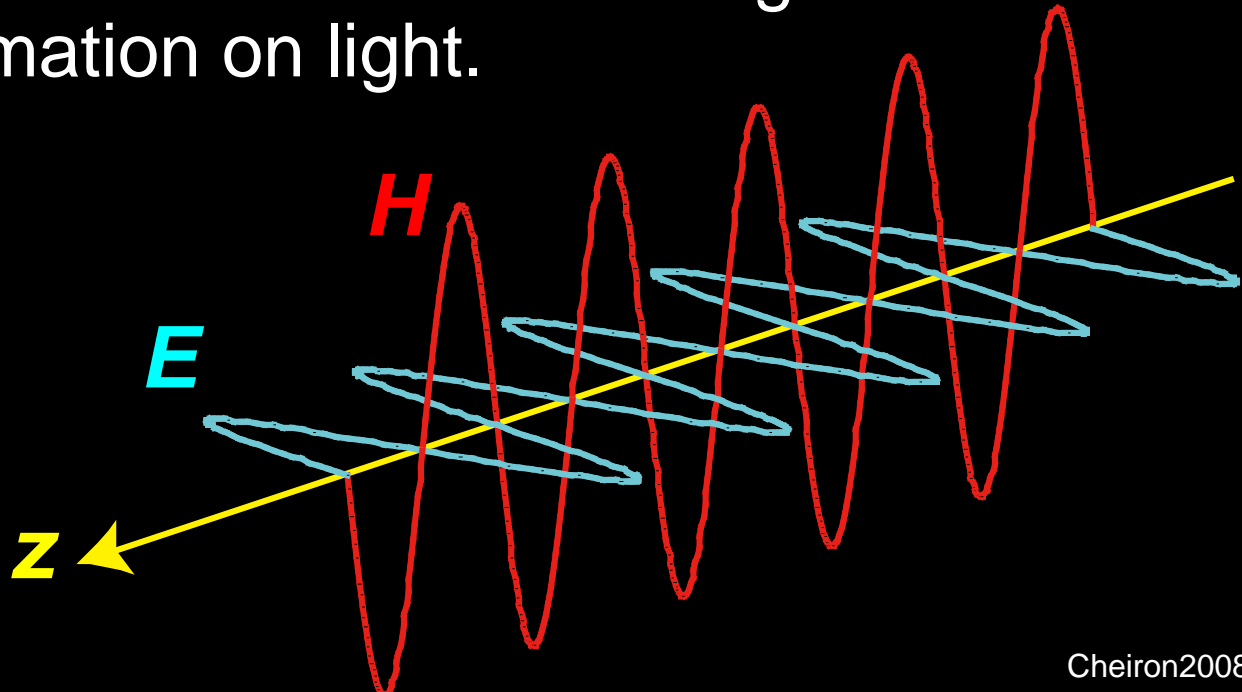
Light as an Electromagnetic Wave

- Electric and magnetic fields can propagate in vacuum as a transverse wave, with a relation

$$H = \frac{1}{\mu_0 c} \hat{z} \times E$$

$\mu_0 c \Rightarrow$ vacuum impedance (377Ω)

- Analysis of the electric field gives all the information on light.



Light as a Photon

- In a quantum theory, light is not only an electromagnetic wave but also a particle, or a photon.
- The energy of a photon with a wavelength of λ is given by

$$E_{\text{photon}} = \hbar\omega = \frac{hc}{\lambda}$$

- For example, a photon with the wavelength of 0.1nm ($1\text{\AA}=10^{-10}\text{m}$) is calculated as 12.4 keV.

Relation Between EM Wave and Photon

E-field in Frequency Domain
 \Rightarrow Complex Amplitude

E-field of Radiation
 in Time Domain

$$\tilde{E}(\omega) = \int E(t) e^{i\omega t}$$

$$N(\omega) = \text{const.} \frac{|\tilde{E}(\omega)|^2}{\hbar\omega}$$

Number of
Photons at ω

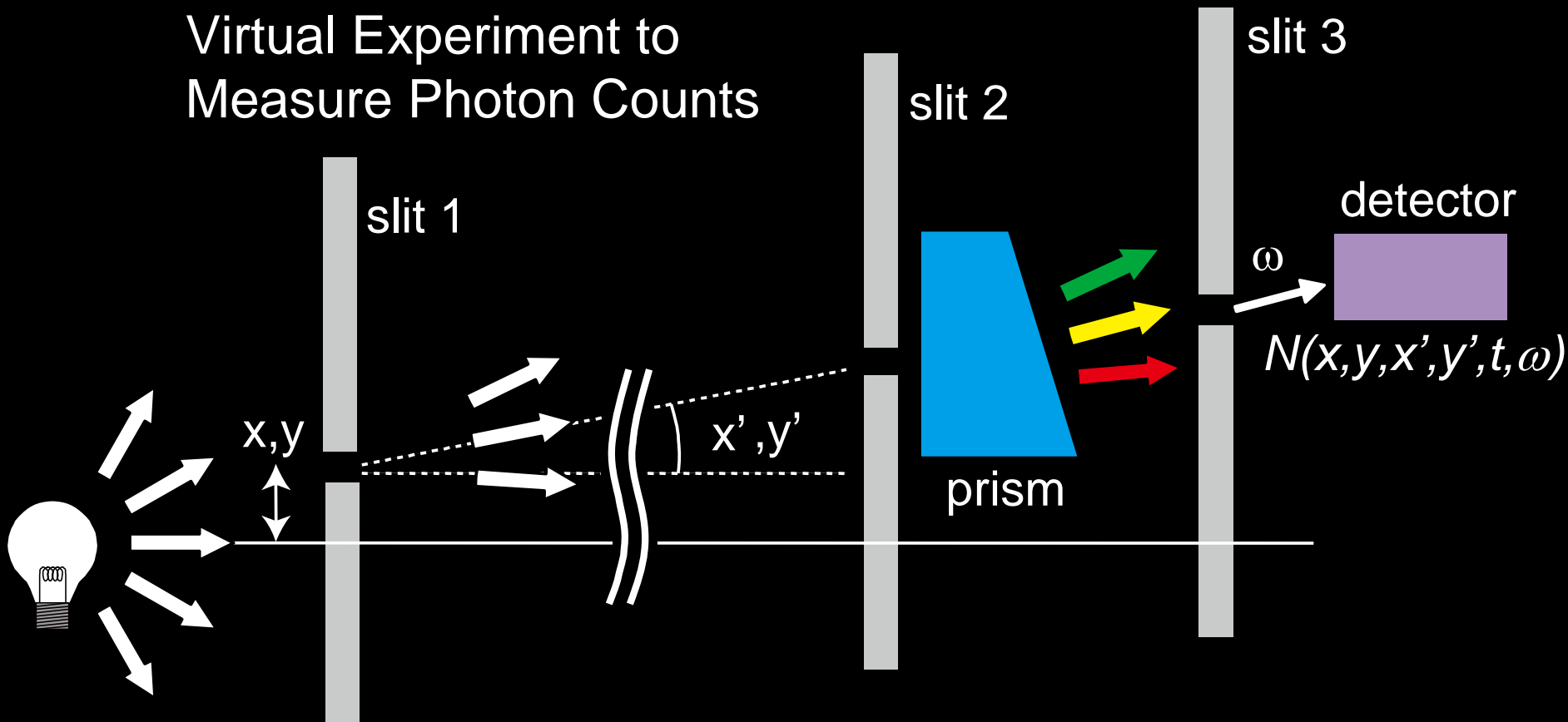
Energy/Single Photon

Fundamentals of Light and SR

- General Description
- **Physical Quantity of Light**
- Uncertainty of Light
- SR: Light from a Moving Electron

Phase Space

Virtual Experiment to Measure Photon Counts



$$n = \frac{N(x, y, x', y', \omega, t)}{\Delta x \Delta y \Delta x' \Delta y' \Delta t \Delta \omega / \omega} \equiv \frac{N}{\Delta \Omega}$$

Av. Photon Density

Volume in 6-D
Phase Space

Brilliance (Brightness)

- Brilliance (photons/sec/mm²/mrad²/0.1%B.W.) is defined as the **photon density in the 6D phase space**, i.e.,

$$B = \lim_{\Delta\Omega \rightarrow 0} n = \frac{d^6 N(x, y, x', y', t, \omega)}{dx dy dx' dy' dt d\omega / \omega}$$

- In practice, $\Delta\Omega$ can never be 0 due to uncertainty of light, thus **brilliance is not a physical quantity that can be actually measured.**

Photon Flux and Flux Density

- Removing the 1st slit gives the angular flux density (photons/sec/mrad²/0.1%B.W), i.e.,

$$\frac{d^2 F}{dx' dy'} = \iint B dx dy$$

- Removing the 1st & 2nd slits gives the total flux (photons/sec/0.1%B.W), i.e.,

$$F = \iiint B dx dy dx' dy'$$

- Estimation of number of photons to be delivered to the sample.

Radiation Power and Power Density

- Removing the 1st & 3rd slits gives the angular power density (W/mrad²), i.e.,

$$\frac{d^2 P}{dx' dy'} = \boxed{10^3 Q_e \hbar} \int \frac{d^2 F}{dx' dy'} d\omega$$

conversion from photons/sec/0.1%B.W. to W

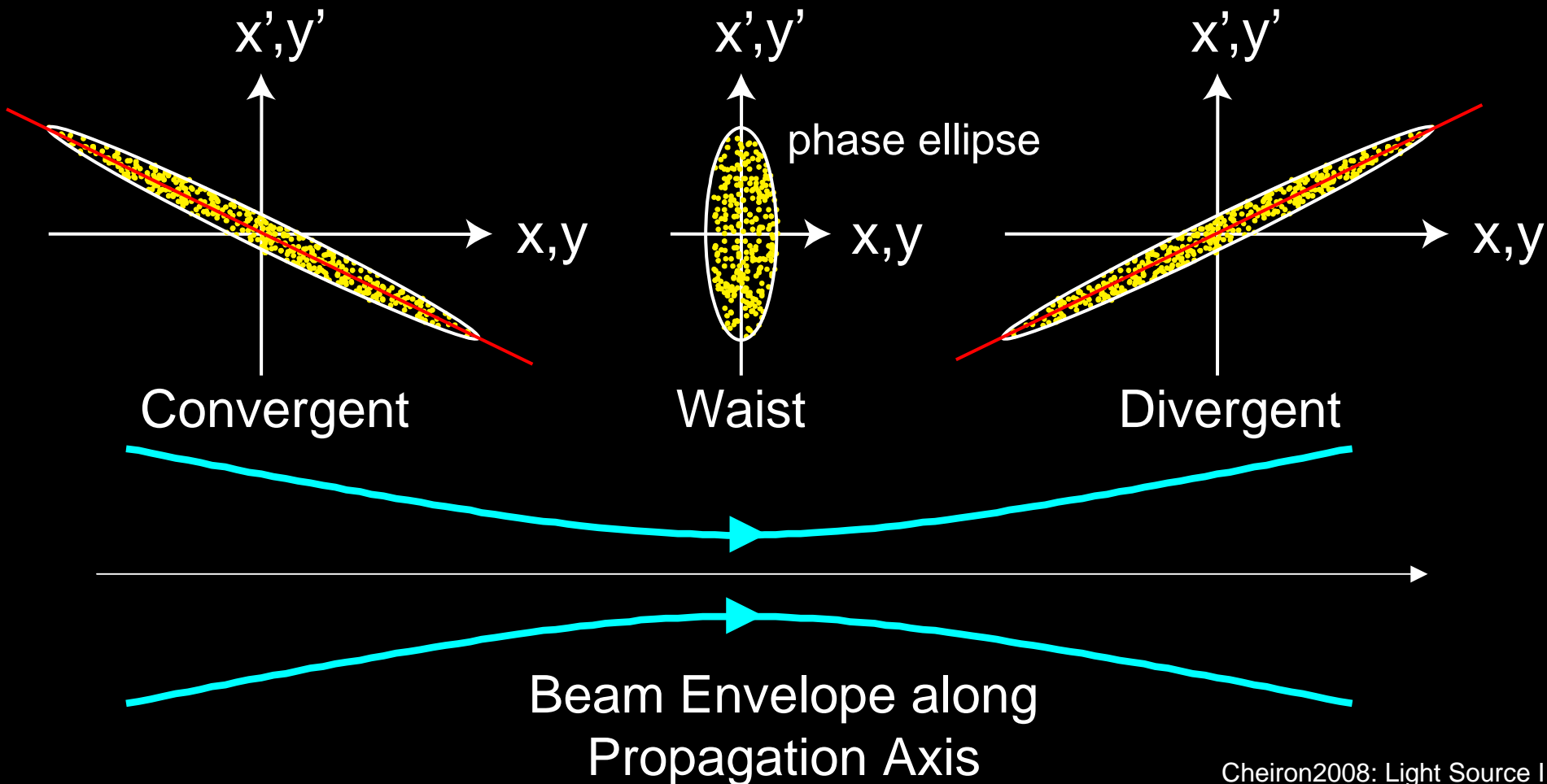
- Removing all the slits gives the total power (W), i.e.,

$$P = 10^3 Q_e \hbar \iiint \frac{d^2 F}{dx' dy'} d\omega dx' dy'$$

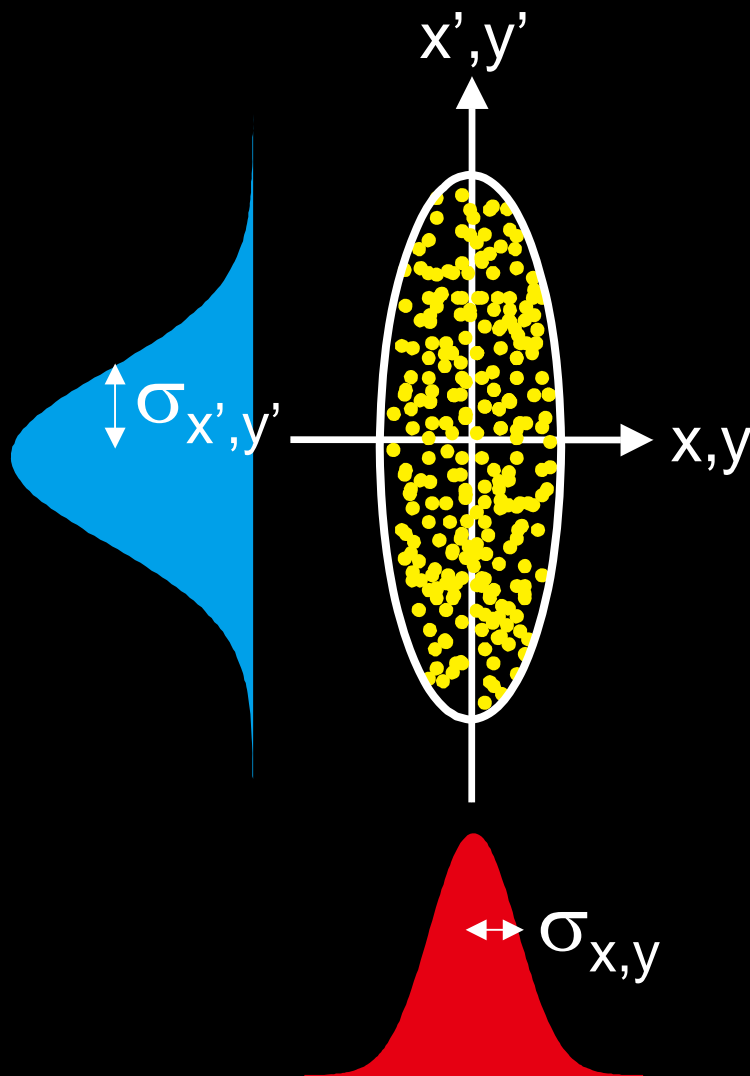
- Estimation of heat load on BL components.

Photons in 4D Phase Space

- Photon distribution in the 4-D phase space at different longitudinal positions.



Beam Size, Divergence, Emittance



- Beam size ($\sigma_{x,y}$) is defined as the beam envelope at the beam waist position.
- Angular divergence ($\sigma_{x',y'}$) is constant along the axis of propagation, as far as no optical elements are present.
- Emittance ($\varepsilon_x, \varepsilon_y$) is defined as $\sigma_{x,y} \times \sigma_{x',y'}$, which is equal to the area of the phase ellipse divided by π .

Estimation of On-Axis Brilliance

- Gaussian Approximation of B

$$B(x, y, x', y') = B_0 \exp \left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{x'^2}{2\sigma_{x'}^2} - \frac{y'^2}{2\sigma_{y'}^2} \right)$$

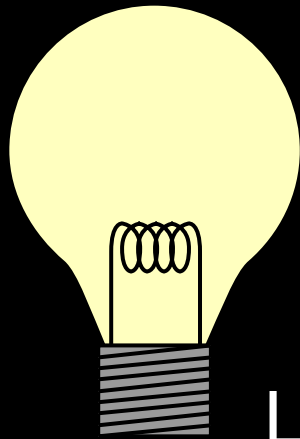
$$\left. \frac{d^2 F}{dx' dy'} \right|_0 = \iint B dx dy = 2\pi\sigma_x\sigma_y B_0$$

$$F = \iiint B dx dy dx' dy' = 4\pi^2\sigma_x\sigma_y\sigma_{x'}\sigma_{y'} B_0$$

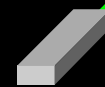
$$B_0 = \left. \frac{d^2 F}{dx' dy'} \right|_0 \frac{1}{2\pi\sigma_x\sigma_y} = \frac{F}{4\pi^2\sigma_x\sigma_y\sigma_{x'}\sigma_{y'}}$$

estimation of on-axis brilliance
with measurable values

Example: Laser Pointer and Light Bulb



Light Bulb



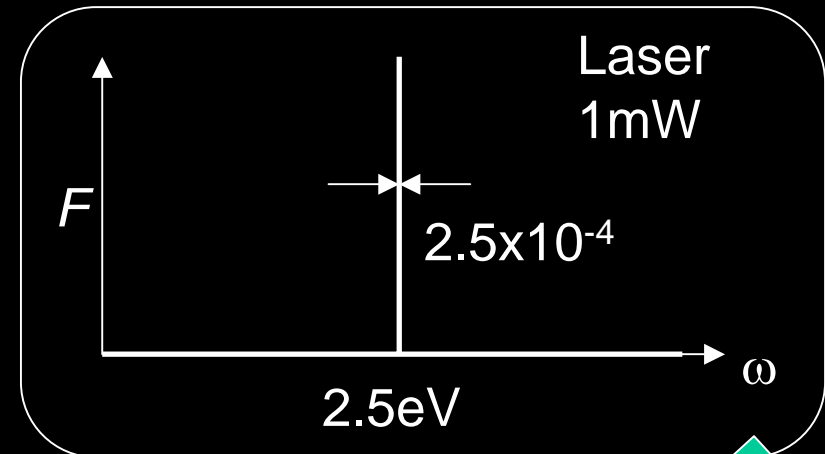
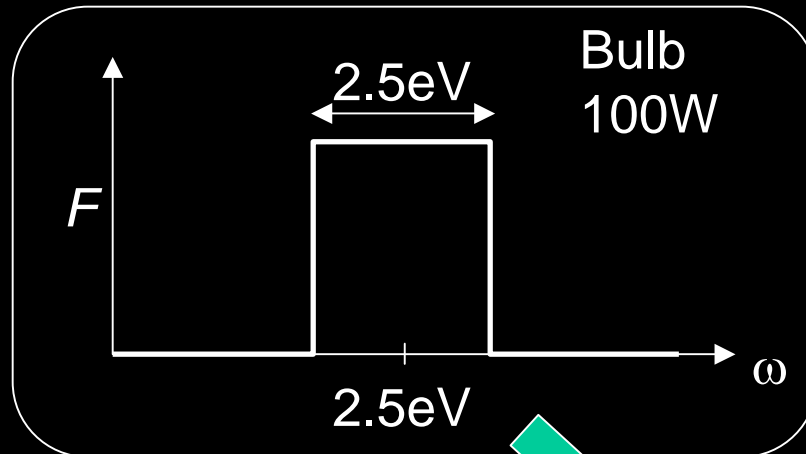
Laser Pointer

Specs of SPring-8

- E = 8GeV
- I = 100mA
- L = 1500m

	Bulb	Laser
$\lambda_r (\hbar\omega_r)$	500nm (2.5eV)	
Band Width:	100 (%)	0.01 (%)
Total Power: W	100 W	1 mW
Angular Divergence	4π rad	1 mrad ²
Beam Size:	1 cm ²	1 mm ²

Example: Photon Flux



$$P = 10^3 Q_e \hbar \int F d\omega$$

	Bulb	Laser
Total Power	100 W	1 mW
Band Width	100 (%)	0.01 (%)
	2.5 eV	2.5x10 ⁻⁴ eV
Flux@500nm	10 ¹⁷	10 ¹⁶

Example: Brilliance

$$B = F / (4\pi^2 \sigma_x \sigma_y \sigma_{x'} \sigma_{y'})$$

	Bulb	Laser
Total Power	100 W	1 mW
Flux@500nm	10^{17}	10^{16}
Angular Divergence	4π rad	1 mrad ²
Beam Size	1 cm ²	1 mm ²
Brilliance	10^7	10^{14}

- **Laser is an ideal light source in terms of brilliance, but ...**
- **No lasers exist in x-ray region**
 - **Synchrotron Radiation**

Fundamentals of Light and SR

- General Description
- Physical Quantity of Light
- **Uncertainty of Light**
- SR: Light from a Moving Electron

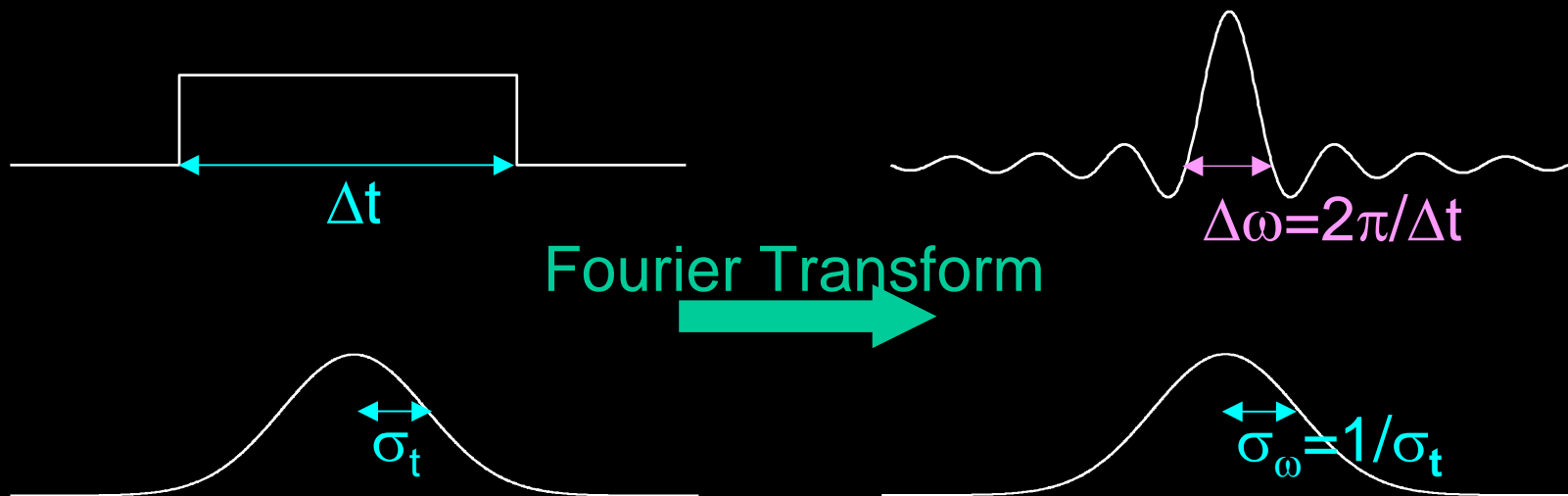
Uncertainty of Light

- The photon distribution in the 6D phase space $(x, y, x', y', t, \omega)$ gives us the full information on the properties of SR.
- Due to wave nature of light, however, we have two uncertainty relations to take care, which are well characterized by the Fourier transform.
- These relations imposes two restrictions on SR, Fourier and Diffraction limits.

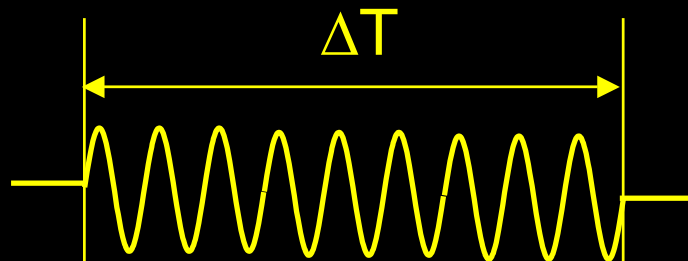
Fourier Transform: Example

Important Fourier Transform in SR Formulae

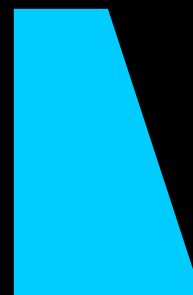
$F(t)$	$f(\omega) = \int_{-\infty}^{\infty} F(t)e^{i\omega t}$
$\begin{cases} 1/\Delta t; & -\Delta t/2 \leq t \leq \Delta t/2 \\ 0; & t < -\Delta t/2, \Delta t/2 < t \end{cases}$	$\frac{\sin \omega \Delta t/2}{\omega \Delta t/2} \equiv \text{sinc}(\omega \Delta t/2)$
$\frac{1}{\sqrt{2\pi\sigma_t}} \exp\left(-\frac{t^2}{2\sigma_t^2}\right)$	$\exp\left(-\frac{\omega^2 \sigma_t^2}{2}\right)$



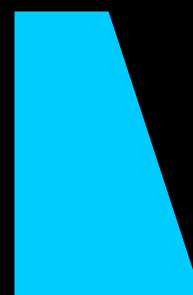
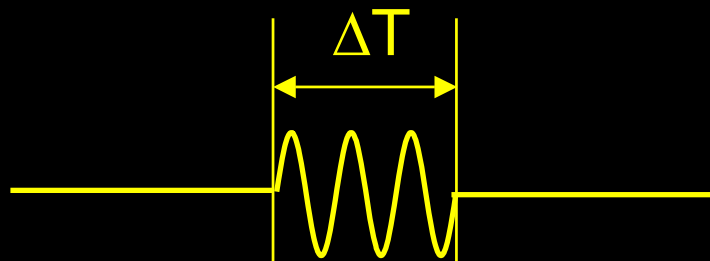
Temporal Fourier Transform



light pulse with duration of ΔT

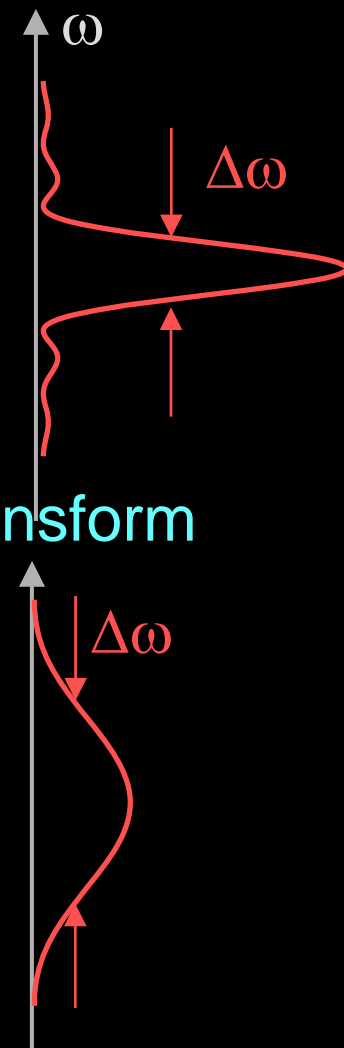


spectrometer = temporal Fourier transform



long Δt narrow $\Delta \omega$
 short Δt wide $\Delta \omega$

↔



Fourier Limit of Light

- Temporal Fourier transform imposes

$$\Delta\omega\Delta t \gtrsim 1$$

- Uncertainty of light in the (ω, t) plane.
- When equality holds, light is said to be Fourier-limited, or temporally coherent.
- Important to understand the spectral properties of SR.

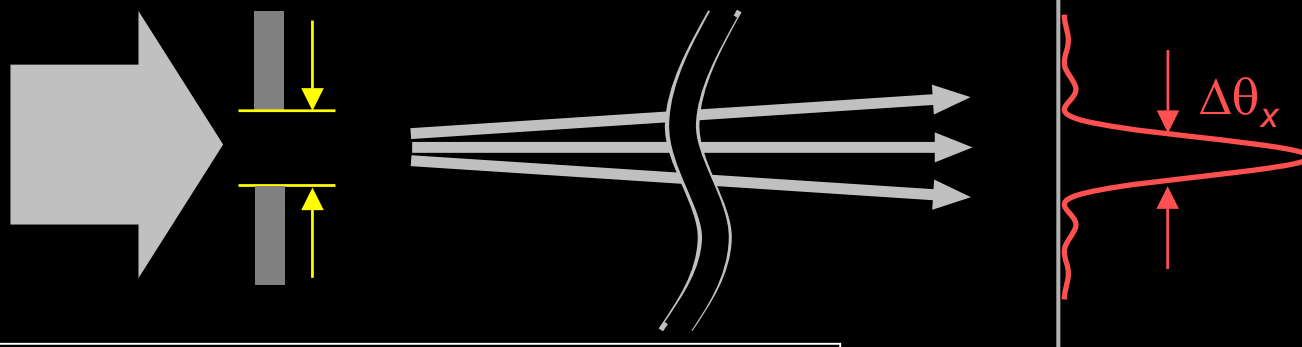
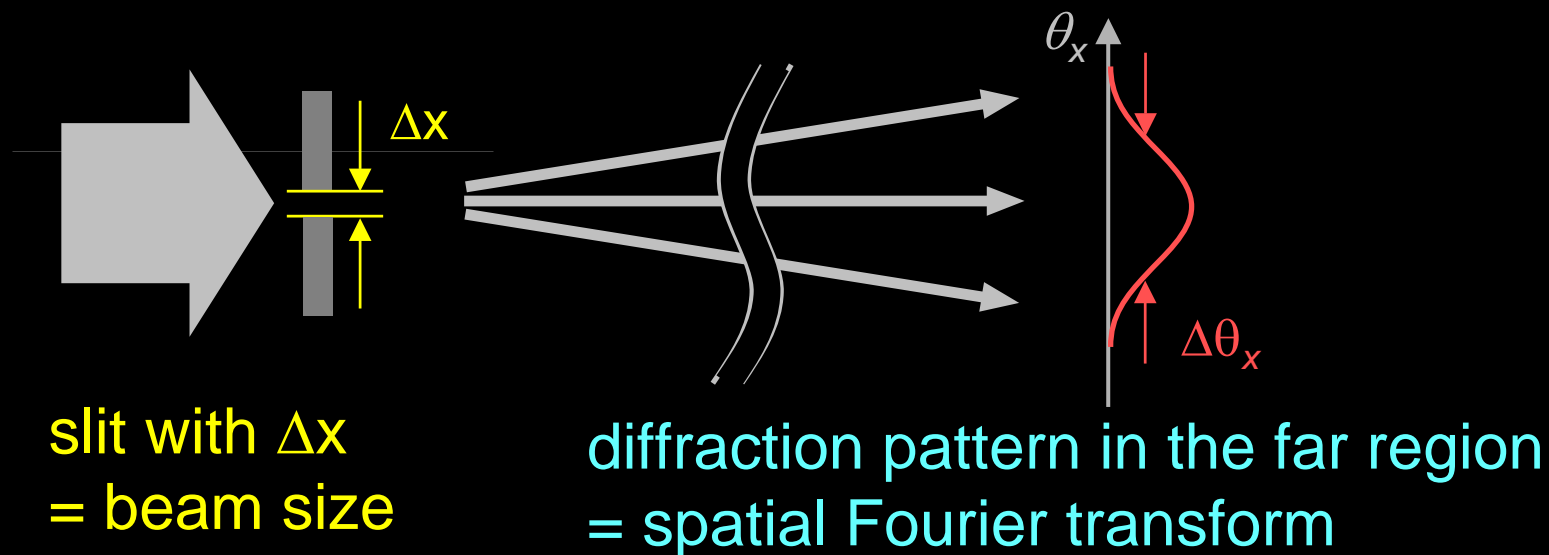
Monochromatic Light

- Ideally, “monochromatic” means

$$\Delta\omega = 0 \text{ \& \ } \Delta t = \infty$$

- In practice, perfectly monochromatic light does not exist, but all kinds of light are partially monochromatic.
- Temporal Fourier transform is to decompose the practical light into components of perfectly monochromatic light.

Spatial Fourier Transform



wide Δx narrow $\Delta\theta_x$
 narrow Δx wide $\Delta\theta_x$

↔

Diffraction Limit of Light (1)

Photon Beam with 1D Gaussian Profile

$$|\tilde{E}(x)|^2 \propto \exp(-x^2/2\sigma_x^2) \quad \text{Photon Spatial Profile}$$

$$\tilde{E}(x) \propto \exp(-x^2/4\sigma_x^2) \quad \text{E-field Spatial Profile}$$

↓ **Fourier transform**

$$\tilde{E}(k_x) \propto \exp(-k_x^2\sigma_x^2) \quad \text{E-field k-vector Profile}$$

$$|\tilde{E}(k_x)|^2 \propto \exp(-2\sigma_x^2 k_x^2) \quad \text{Photon k-vector Profile}$$

$$\equiv \exp(-\theta_x^2/2\sigma_{x'}^2) \quad \text{Photon Angular Profile}$$

↓ $k_x = (2\pi/\lambda)\theta_x$

$$\sigma_{x'}\sigma_x = \frac{\lambda}{4\pi} \quad \text{Natural emittance of light with wavelength } \lambda$$

Diffraction Limit of Light (2)

- Spatial Fourier transform imposes

$$\sigma_{x,y}\sigma_{x',y'} \gtrsim \lambda$$

- Uncertainty of light in the (x,x',y,y') plane.
- When equality holds, light is said to be diffraction-limited, or spatially coherent.
- Important to derive beam size and angular divergence SR.

Plane Wave

- Ideally, “plane wave” means

$$\sigma_{x',y'} = 0 \ \& \ \sigma_{x,y} = \infty$$

- In practice, perfectly plane wave does not exist, but all kinds of light are partially plane-wave-like.
- Spatial Fourier transform is to decompose the practical light into components of perfectly plane wave.

Fundamentals of Light and SR

- General Description
- Physical Quantity of Light
- Uncertainty of Light
- **SR: Light from a Moving Electron**

SR: Light from a Moving Electron

- Unlike the ordinary light source (sun, light bulb,...), the light emitter of SR (electron) is ultra-relativistic.
- The characteristics of SR is thus quite different due to relativistic effects.
- What we have to take care is:
 1. Speed-of-light limit
 2. Squeezing of light pulse
 3. Conversion of the emission angles

Speed-of-Light Limit

Within the framework of relativity, the velocity of an electron never exceeds the speed of light.

$$v/c = \beta = \sqrt{1 - \gamma^{-2}}$$

$$\sim 1 - \frac{1}{2\gamma^2}$$

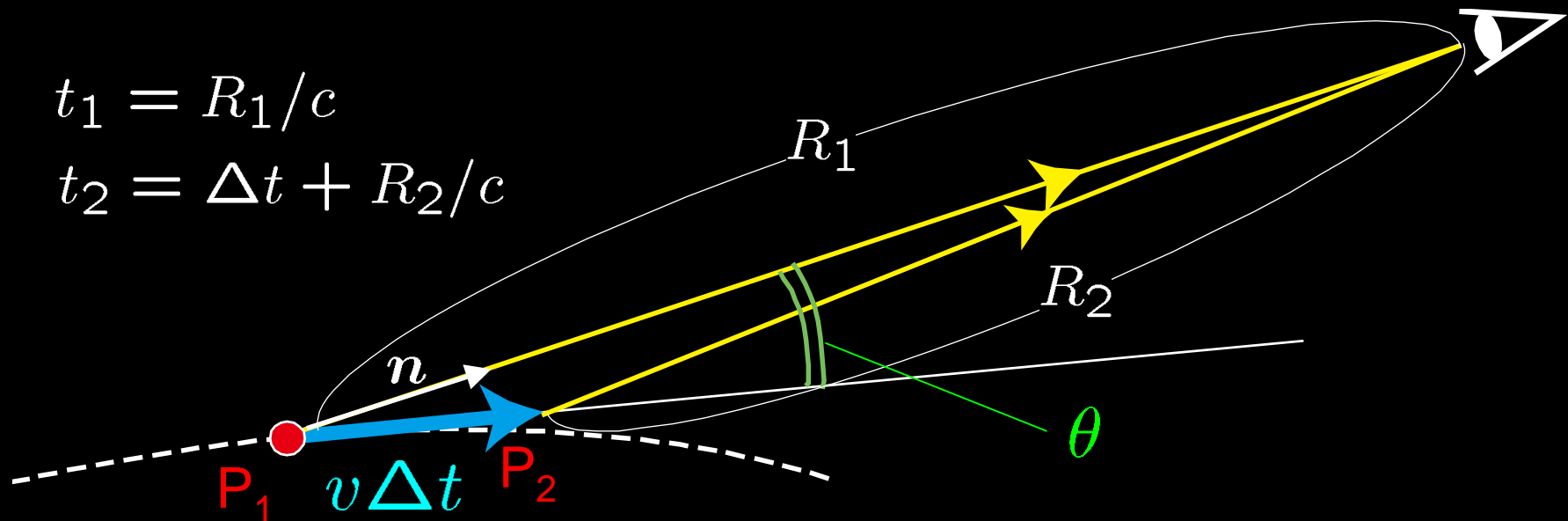
$$\gamma = \frac{E}{mc^2}$$

Energy	β
1MeV	0.941
10MeV	0.9988
100MeV	0.999987
8GeV	0.999999998

:Lorentz Factor

(relative electron energy, $mc^2=0.511\text{ MeV}$)

Squeezing of Light Pulse Duration



$$t_1 = R_1/c$$

$$t_2 = \Delta t + R_2/c$$

$$R_2 = \sqrt{(R_1)^2 + (v\Delta t)^2 - 2R_1v\Delta t \cos \theta}$$

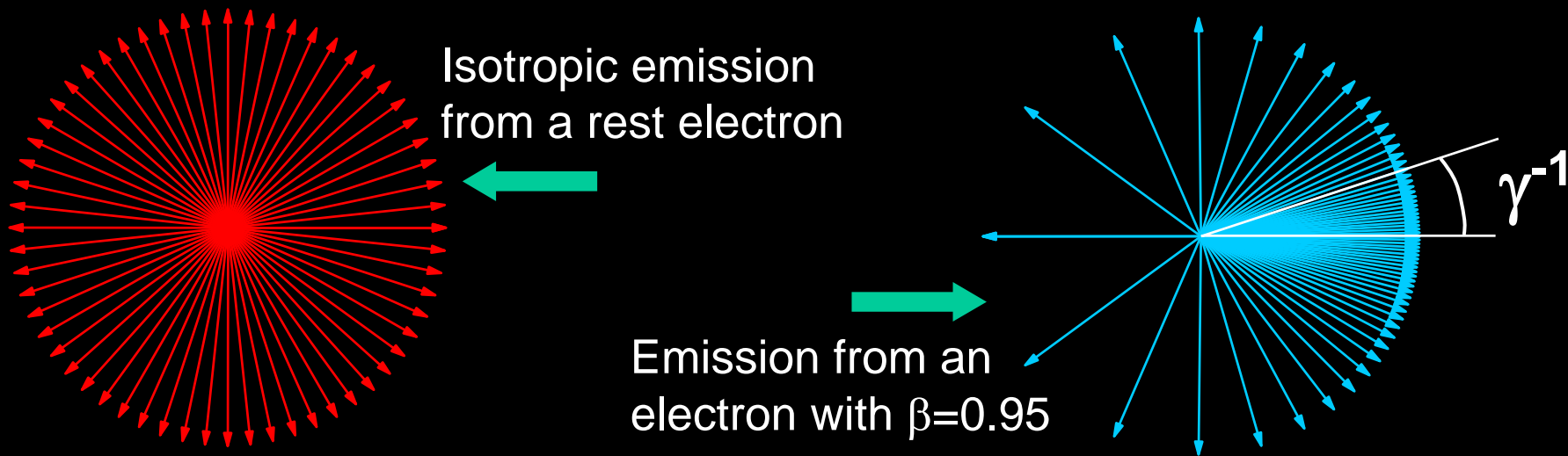
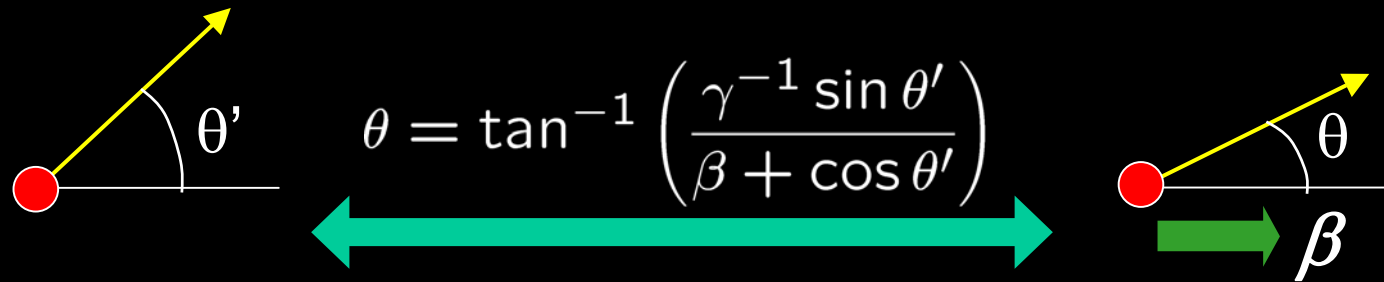
$$\sim R_1 - (v \cdot n)\Delta t$$

$$\Delta\tau = t_2 - t_1 = \Delta t + R_2/c - R_1/c$$

$$= \Delta t \boxed{(1 - \beta \cdot n)} = \boxed{\frac{\Delta t}{2\gamma^2}} \quad \gamma \gg 1, \theta = 0$$

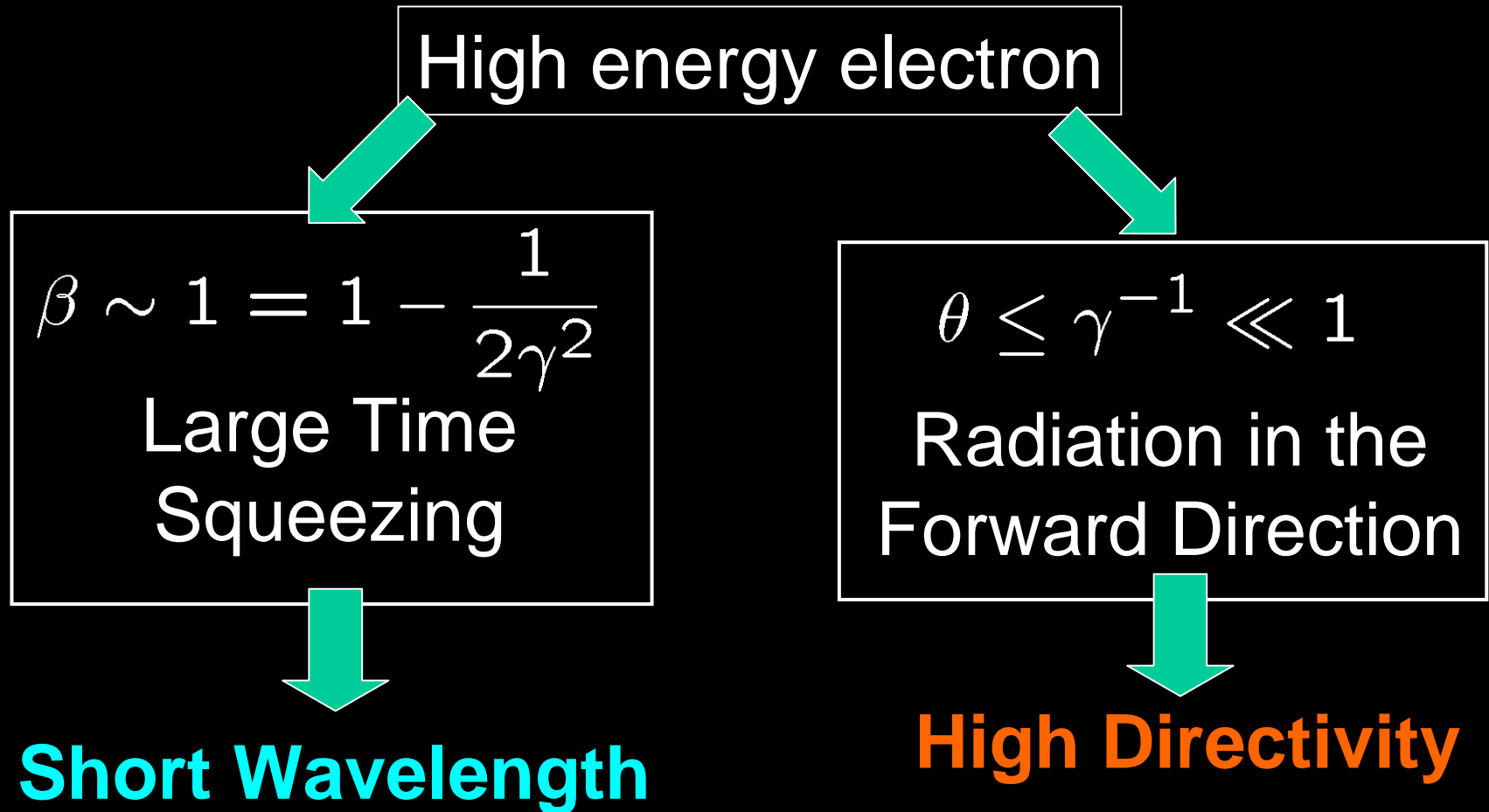
time squeezing

Conversion of Emission Angles



Light emitted from a moving object ($\beta \sim 1$) concentrates within γ^{-1}

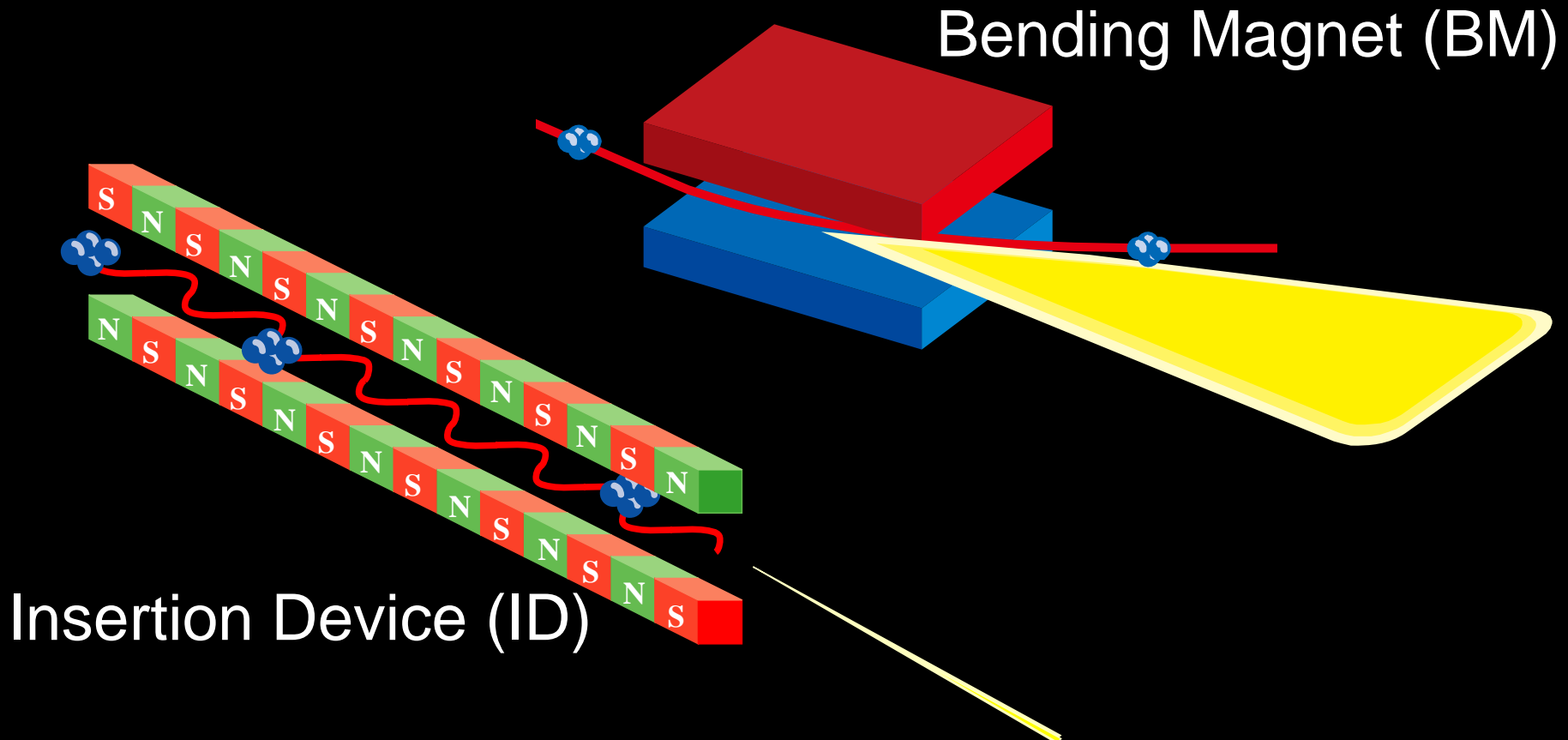
SR from a High-Energy Electron



Overview of SR Light Source

What is SR Light Source?

Magnets to deflect the electron beam and generate SR.



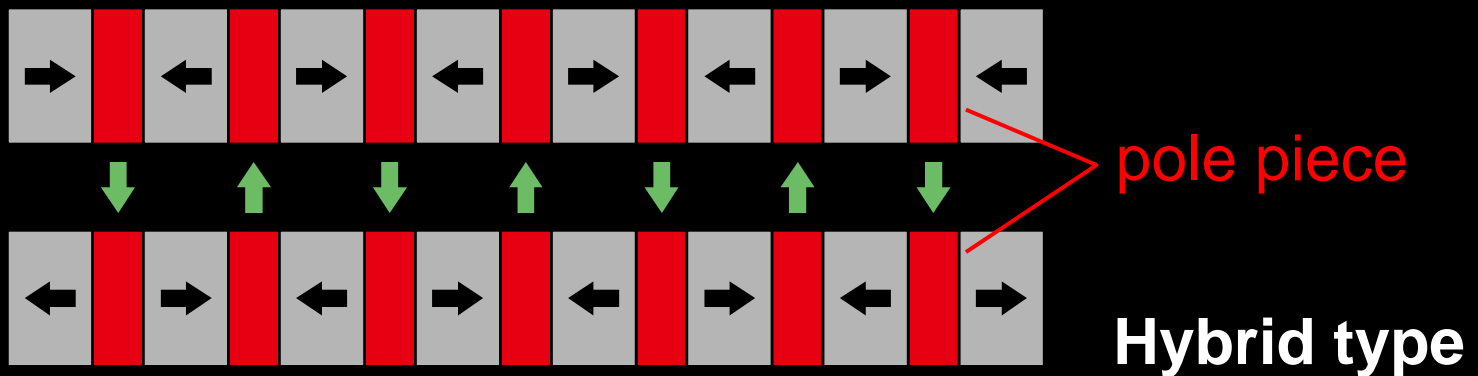
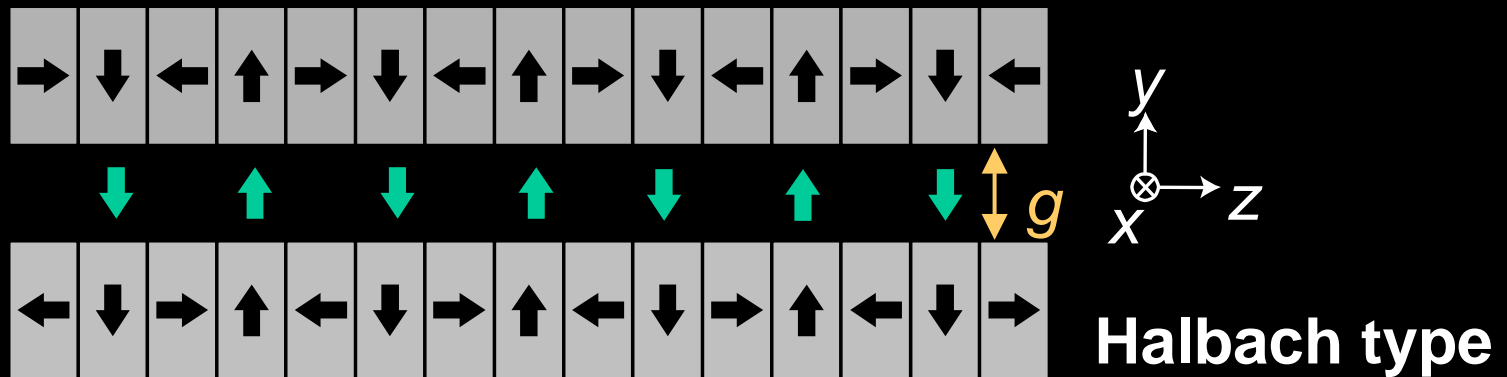
Bending Magnet

- One of the accelerator components in the storage ring.
- Generate uniform field to guide the electron beam into a circular orbit.
- EMs combined with highly-stable power supplies are adopted in most BMs due to stringent requirement on field quality and stability.
- Superconducting magnets are used in a few facilities in pursuit of harder x rays.

Insertion Device

- Installed (inserted) into the straight section of the storage ring between two adjacent BMs.
- Generate a periodic magnetic field to let the injected electron beam move along a periodic trajectory.
- Most IDs are composed of PMs, while EMs are used for special use such as helicity switching.
- Classified into **wigglers** and **undulators**.

Magnetic Circuit of IDs

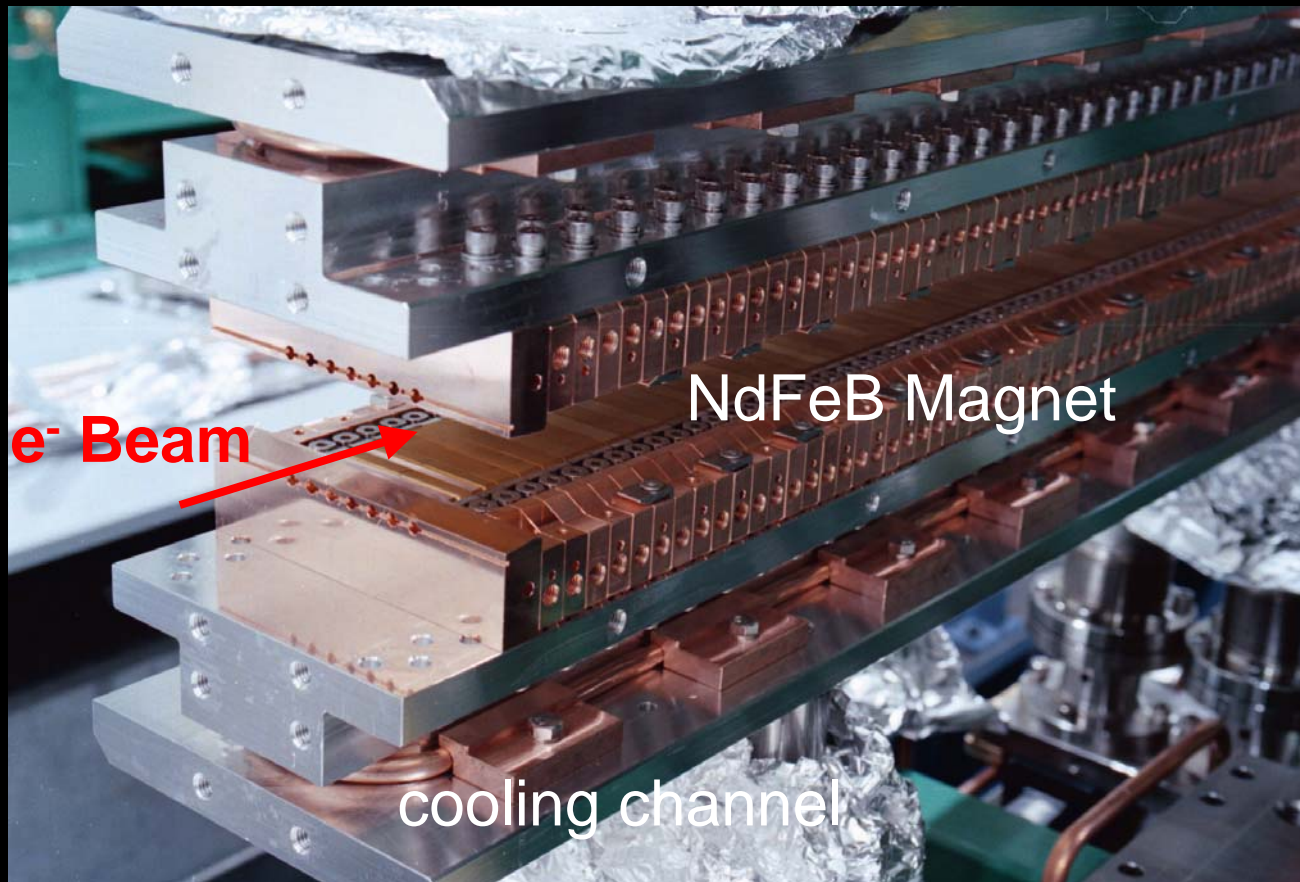


In each type, a sinusoidal magnetic field is obtained:

$$B_y(z) \sim B_0(B_r, g/\lambda_u) \sin\left(\frac{2\pi z}{\lambda_u}\right)$$

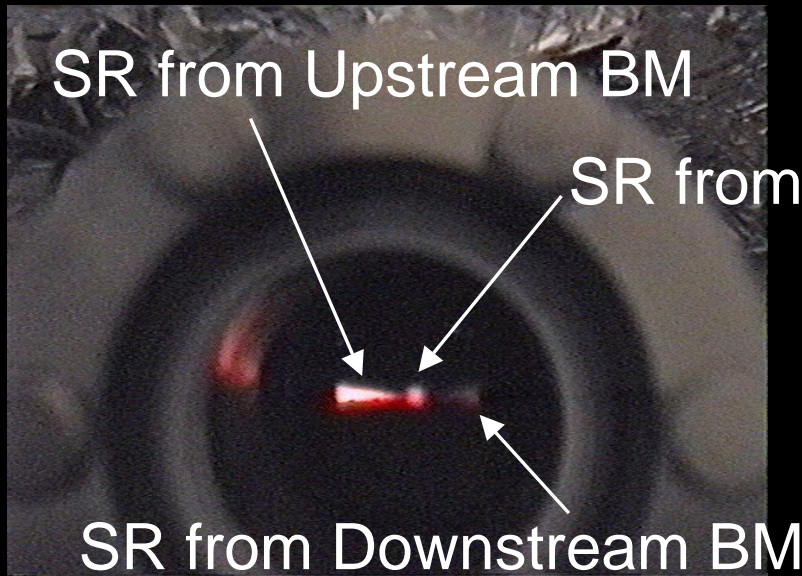
Example of ID Magnets

Halbach-type Magnet Array for SPring-8 Standard Undulators

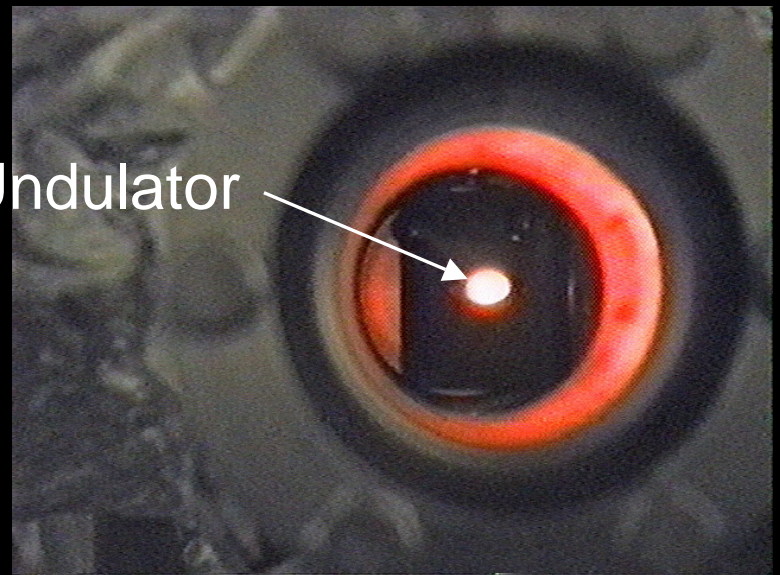


Example of SR Image

BL41XU@SP-8, First Image of SR
at Fluorescent Screen (<0.1mA)

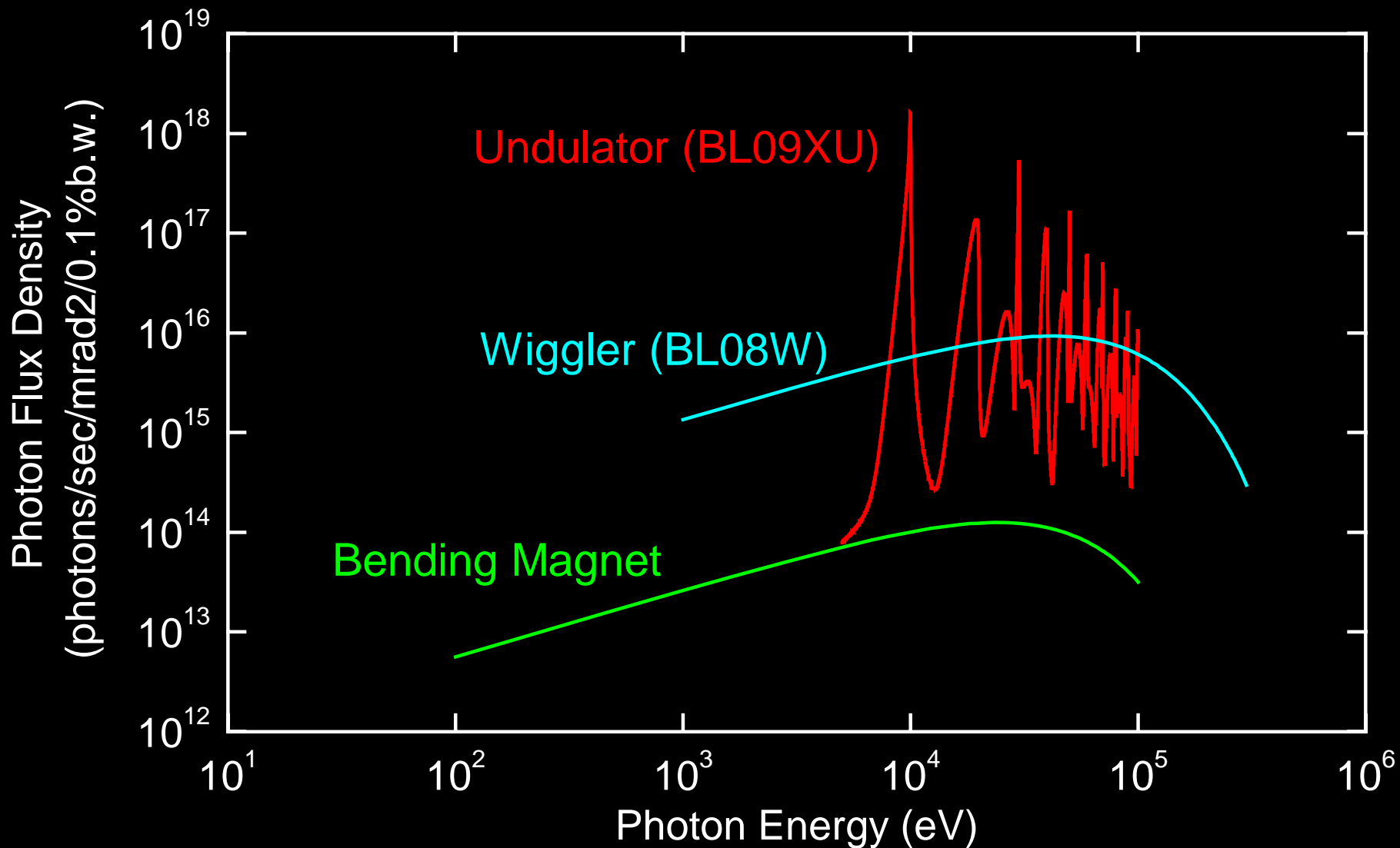


Undulator Gap = 50 mm



Undulator Gap = 20 mm

Comparison of Light Sources

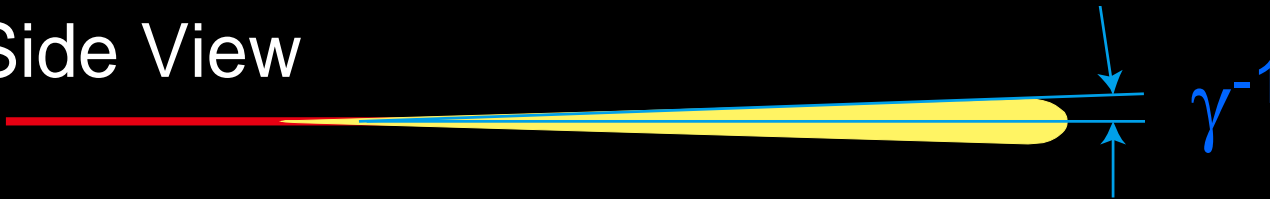


Characteristics of SR (1)

- Radiation from BMs
- Radiation from Wigglers

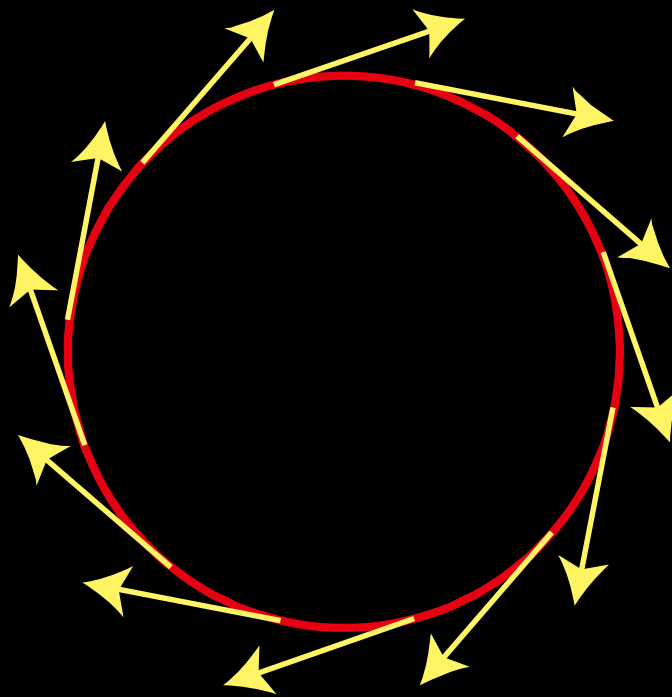
Directivity of BM Radiation

Side View



↑ High directivity in the vertical plane
($\sigma_y \sim \gamma^{-1} \sim 64 \mu\text{rad} @ \text{SP-8}$)

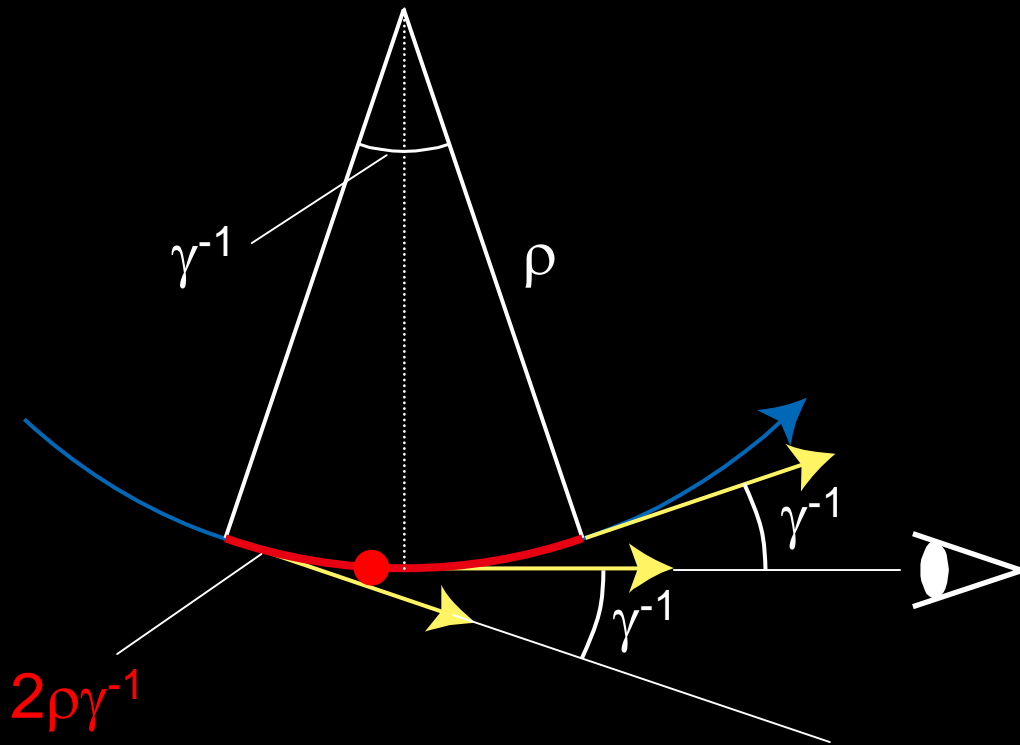
← Isotropic in the orbital plane



Top View

2-dimensional directivity

Spectrum of BM Radiation (1)



Major contribution of radiation is from the portion painted red

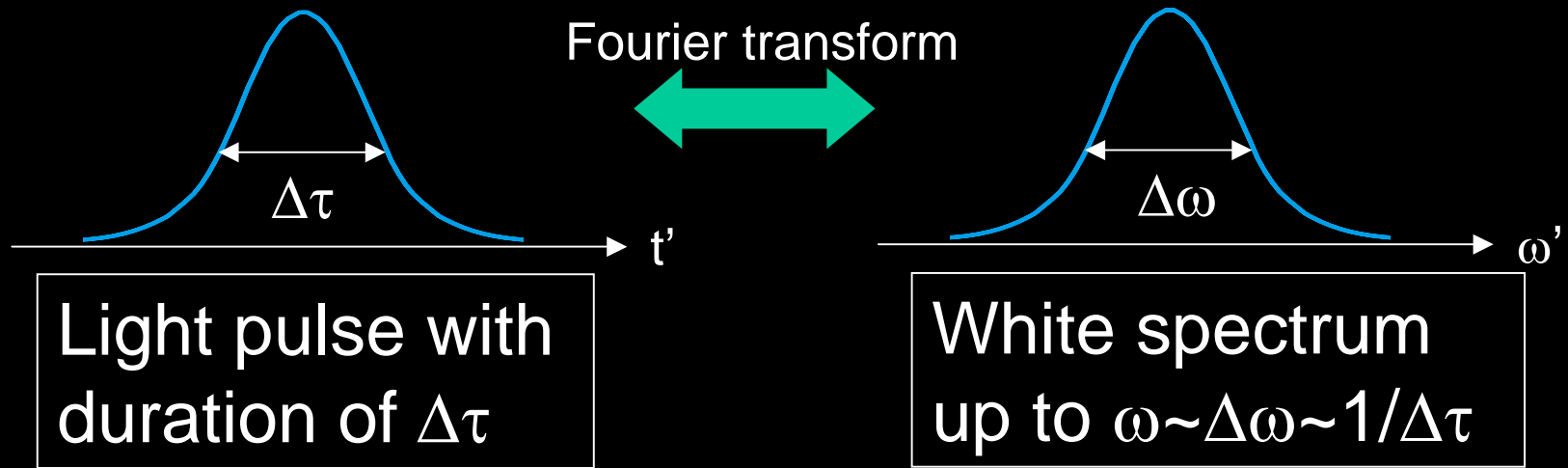
Pulse duration for e^-
 $\Delta t = 2\rho\gamma^{-1}/c$

squeezing

Pulse duration for observer

$$\Delta\tau = \frac{\Delta t}{2\gamma^2} = \frac{\rho}{\gamma^3 c}$$

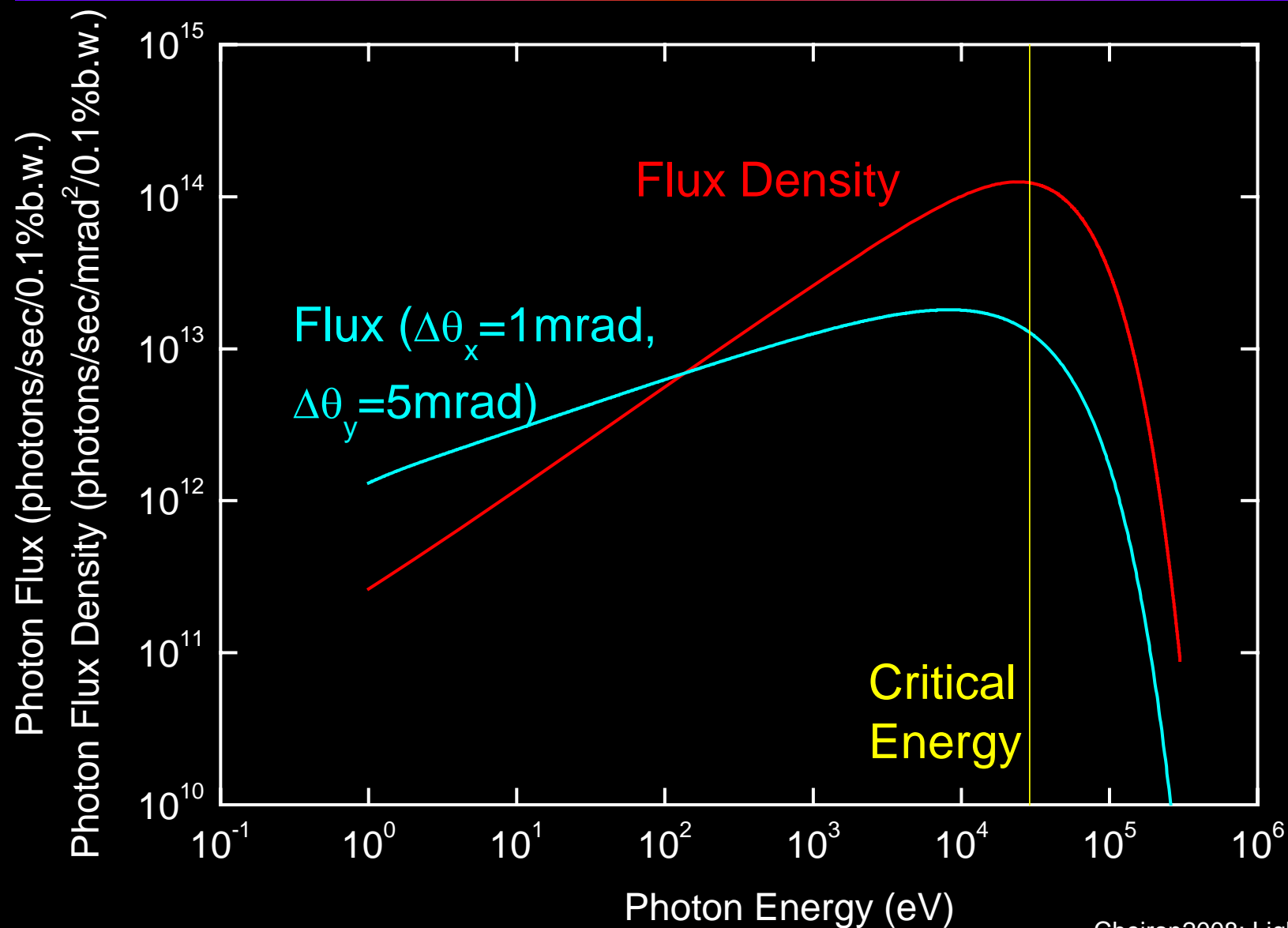
Spectrum of BM Radiation (2)



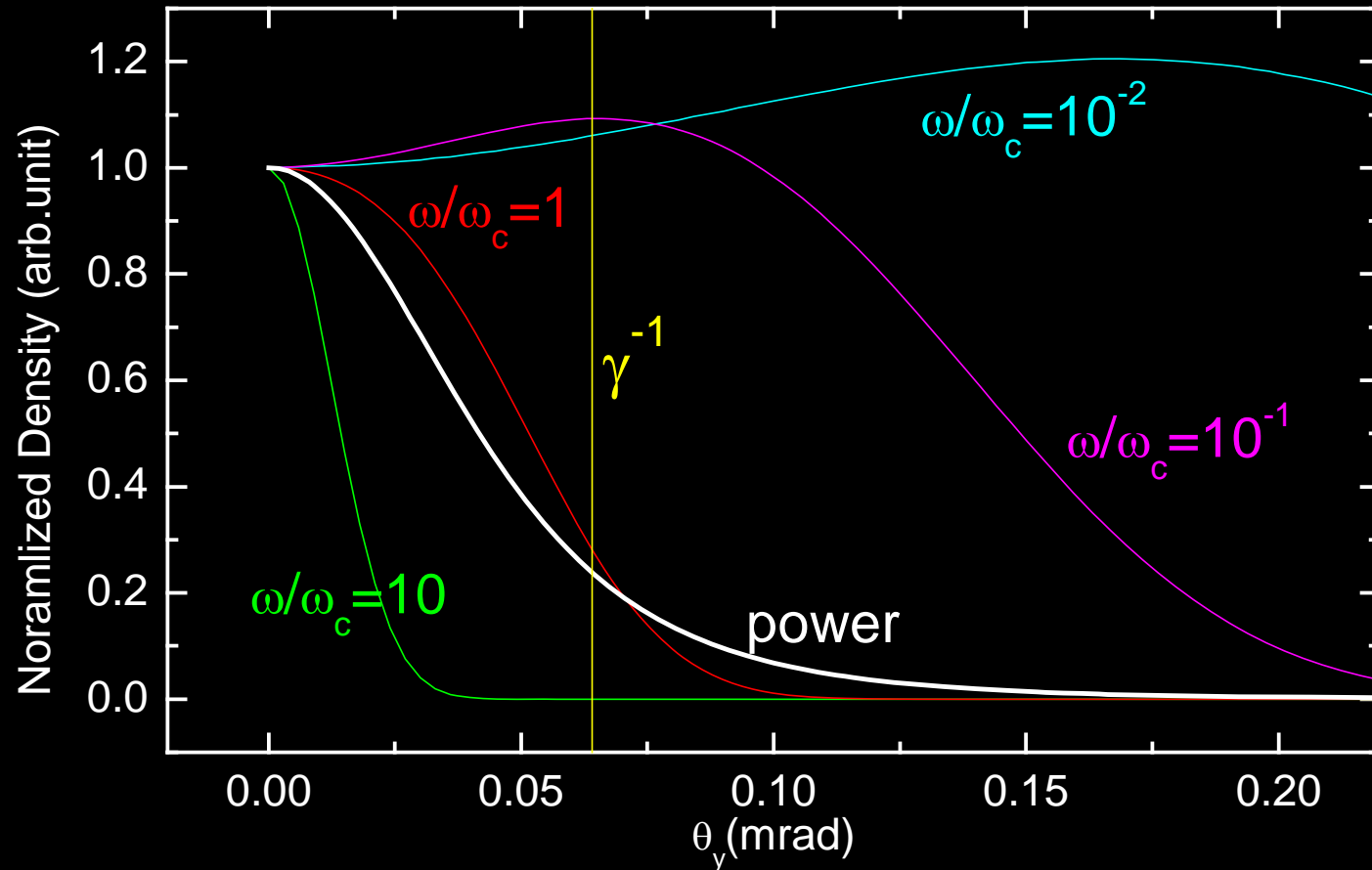
- By definition, $\omega_c = (3/2)\Delta\tau = 3\gamma^3 c / 2\rho$ is called “critical frequency” of SR, which gives a criterion of the maximum energy of SR from a BM.
- In practical units,

$$\hbar\omega_c(\text{keV}) = 0.665 E_e^2(\text{GeV}) B(\text{T})$$

Example of Spectrum



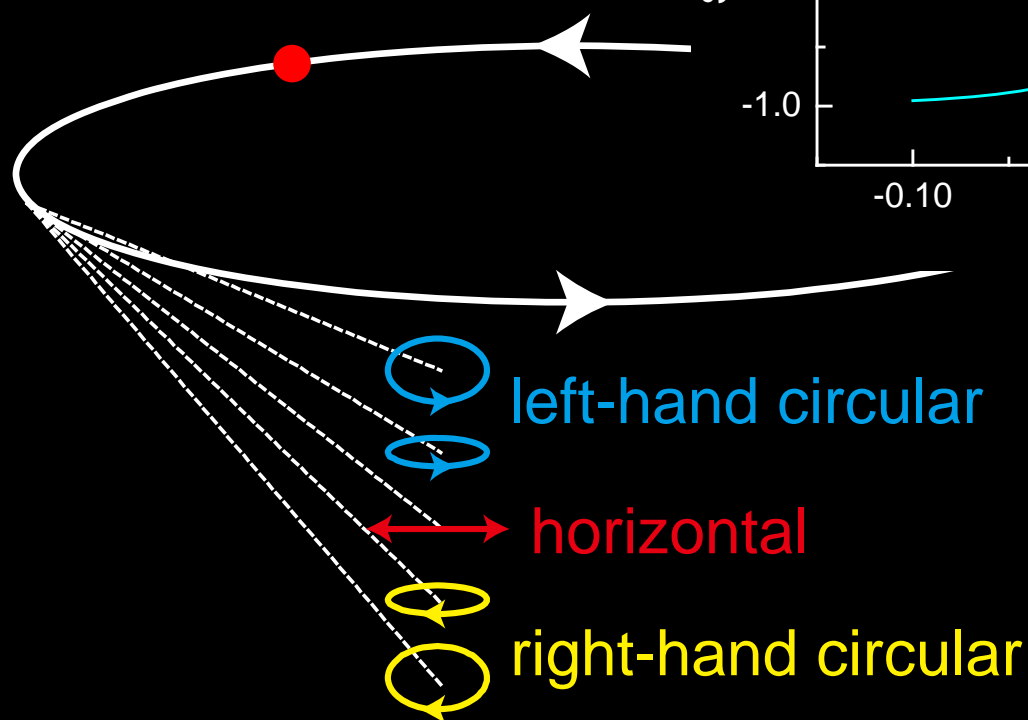
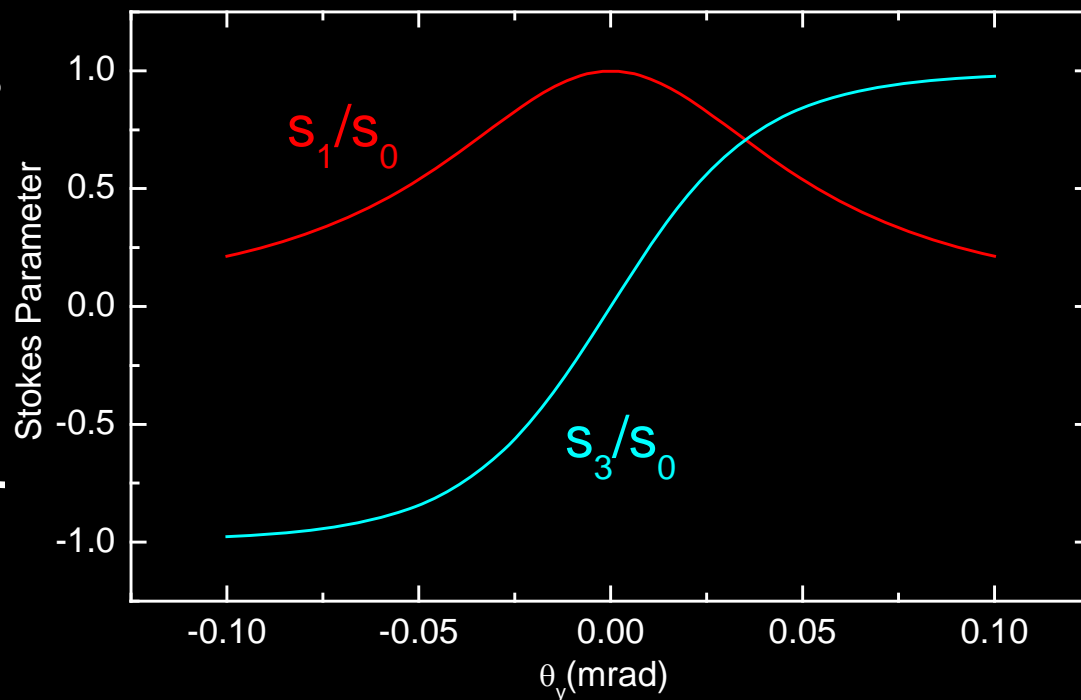
Angular Profile of BM Radiation



- power profile \sim flux profile @ $\omega/\omega_c = 1$
- larger angular divergence for lower energy

Polarization of BM Radiation

Stokes parameters
of BM radiation
along vertical axis



Polarization state
reflects the apparent
motion of electron.

Characteristics of SR (1)

- Radiation from BMs
- Radiation from W wigglers

Wiggler Radiation

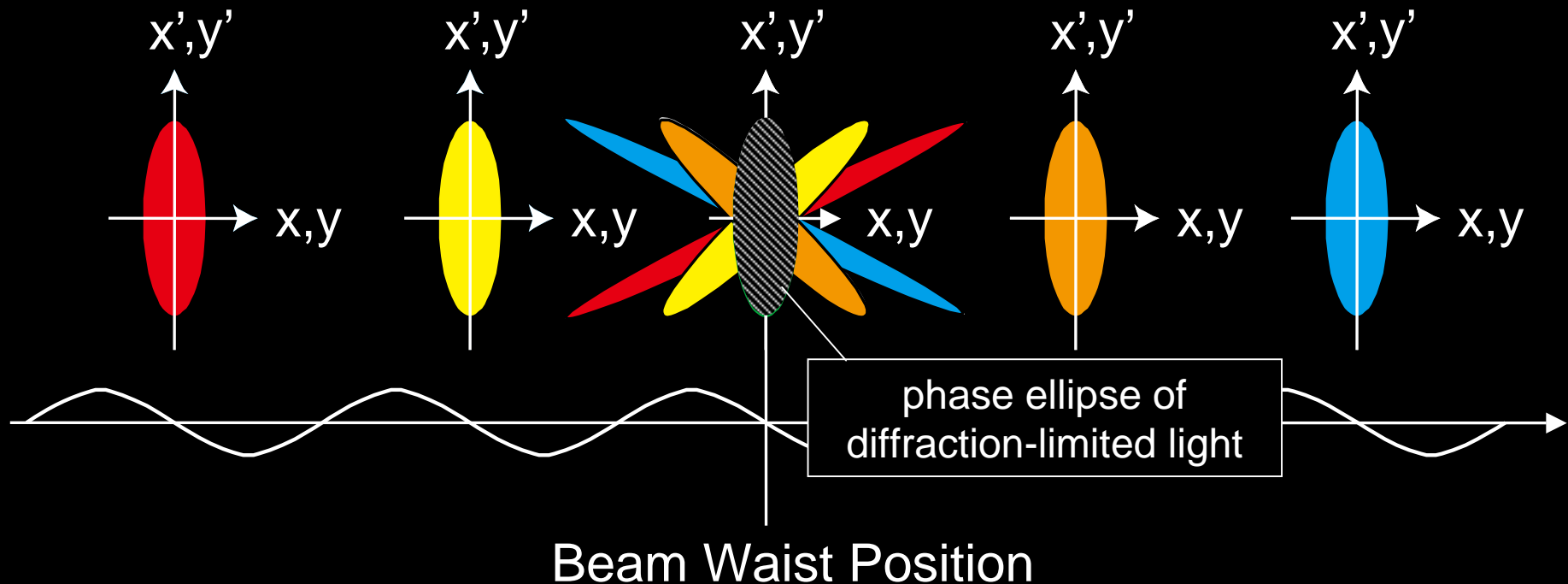
- Wiggler radiation (WR) is regarded as **incoherent sum of SR** at each position.
 - Summation as photons within the framework of **geometrical optics**.

$$\text{Flux: } F_W \sim 2NF_{BM}$$

$$\text{Emittance: } \sigma_{x',y'} \times \sigma_{x,y} \gg \lambda/4\pi$$

$$\text{Brilliance: } B_W \ll 2NB_{BM}$$

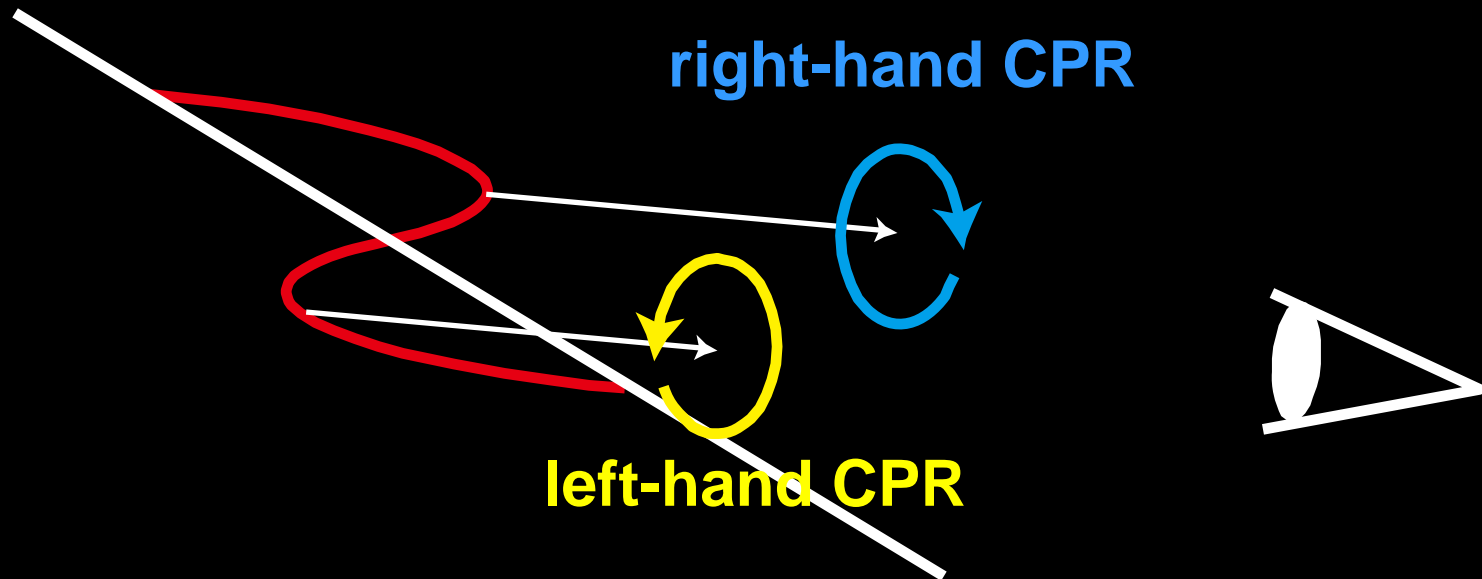
Photon Distribution in Phase Space



- Larger N results in larger area of photon distribution in the phase space, i.e., larger emittance.
- B does not linearly depend on N

Polarization

- No circular polarized radiation (CPR) is observed unlike the BM radiation even off axis, due to cancellation of CPR components.



- EMPW is a special wiggler to utilize CPR by introducing a vertical motion.