Ring Accelerator Physics for Non Accelerator Specialist

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Contents

1. Introduction
2. Light source properties vs electron beam performance
3. Stochasticity of photoemission
4. Distribution of circulating electron beam
5. Current limiting factors
6. Beam stability
7. New idea towards higher beam performances

Introduction (1)

Even any member in an experiment-related division or BL scientists, who are not the accelerator specialists, should understand the relation between the SR properties and electron beam performances,

to keep the present experimental condition,
to improve the condition as much as possible, and
to challenge new experiments by using the advanced SR.

Introduction (2)

Filter

Pumping

Chemical treatment

SR light

Sample

What determines the brilliance ?

Introduction (3)

Mutual understanding between accelerator physicists and experimental division staff through the communication surely makes the SR source performance better for users.

Light source properties vs electron beam performance(1)

Main devices to supply SR to users are “undulators”, which are installed in magnet-free straight sections in 3rd-generation SR sources.

2. Light source properties vs electron beam performance(2)

In an undulator, radiation from a single electron at each undulation interferes with each other.

Spectral narrowing by resonant condition for a single electron is

$$\lambda_1 = \frac{\lambda_u}{2\gamma^2}(1+K^2/2+\gamma^2\theta^2)$$

where K and γ are a deflecting parameter and relative energy.

2. Light source properties vs electron beam performance(3)

Q: Why does the radiation concentrate within the γ cone ?

A: The electron can run after the emitted light with the almost light speed only in this limited angle.

retarded time

$$t = t' + \frac{R(t')}{c}$$

present time

Time squeezing factor $\kappa(t') = \dfrac{\Delta t}{\Delta t'}$

$\kappa(t')$ small → large acceleration → strong radiation

$$\kappa(t') = \frac{\Delta t}{\Delta t'} = 1 - \vec{n}(t') \cdot \vec{\beta}(t')$$

$$\kappa(t') \approx \frac{1}{2}\left(\frac{1}{\gamma^2} + \theta^2\right)$$

2. Light source properties vs electron beam performance(4)

Q: Why does the radiation concentrate within the γ cone ?

A: The electron runs after the emitted light with the almost light speed only in this limited angle.

$$\kappa(t') \approx \frac{1}{2}\left(\frac{1}{\gamma^2} + \theta^2\right)$$

$\theta < 1/\gamma \rightarrow \kappa \sim 1/\gamma^2$

Intense radiation is emitted in this narrow angle

2. Light source properties vs electron beam performance(5)

If electron beam were point-like without spatial divergence and , all electrons could have no angular divergence, radiations were coherent !

Real radiation properties are obtained by convoluting radiations from all N electrons

In the real world N electrons are distributed in a phase space, never degenerate on the same point.

2. Light source properties vs electron beam performance(6)

In order to obtain the same resonance condition over most of circulating electrons distributed, sufficiently small beam emittance is required.

2. Light source properties vs electron beam performance(7)

In order to obtain the same resonance condition over most of circulating electrons distributed, sufficiently small beam energy spread is required.

$$\lambda_1 = \frac{\lambda_u}{2(\gamma+\Delta\gamma)^2}(1+K^2/2+\gamma^2\theta^2)$$

N_u:undulator period number

$$\Delta\lambda_1 \sim 2\lambda_1 \langle\Delta\gamma^2\rangle$$

Spectral broadening by the energy spread is much less than intrinsic broadening ~$1/N_u$.

2. Light source properties vs electron beam performance(8)

$$B \propto \frac{I_b \cdot N_u^2 \cdot \gamma^2 \cdot F_n(K, \delta)}{\sigma_x \sigma_{x'} \sigma_y \sigma_{y'}},$$

B: Brilliance (phs/sec/mm^2/mrad^2/100mA)

I_b: Beam current (mA)

N_u: Undulator period number

σ_x, σ_y: Horizontal and vertical beam sizes (m)

$\sigma_{x'}, \sigma_{y'}$: Horizontal and vertical angular divergence (rad)

δ: Beam energy spread

K :Deflection parameter

2. Light source properties vs electron beam performance(9)

$$\sigma_x=\sqrt{\sigma_p^2+\beta_x\varepsilon_x+\eta_x^2\delta^2},\ \sigma_{x'}=\sqrt{\sigma_{p'}^2+\gamma_x\varepsilon_x+\eta'^2_x\delta^2}\ ,$$

$$\sigma_y=\sqrt{\sigma_p^2+\beta_y\varepsilon_y+\eta_y^2\delta^2},\ \sigma_{y'}=\sqrt{\sigma_{p'}^2+\gamma_y\varepsilon_y+\eta'^2_y\delta^2}\ ,$$

$$\alpha_{x,y}=\frac{-d\beta_{x,y}}{ds},\quad \gamma_{x,y}=\frac{1+\alpha_{x,y}^2}{\beta_{x,y}},\quad \eta'_{x,y}=\frac{d\eta_{x,y}}{ds}.$$

Photon

Electron emittance

Electron energy spread

$\varepsilon_x,\varepsilon_y$: Horizontal and vertical emittance (m•rad)

$\beta_{x,y}$: Horizontal and vertical betatoron functions at ID

$\eta_{x,y}$: Horizontal and vertical dispersion functions at ID

$\sigma_p,\sigma_{p'}$: Spatial and angular divergence of photon beam

3. Stochasticity of photoemission(1)

The photo-emission process is **not continuous**, in other words, **quantized process**. So, the emission position, the number of the emission photons, and the emission photon energy have **fluctuations**.

This stochasticity (random fluctuation) causes finite spread of the circulating electron beam in the 6D phase space. The density distribution is generally Gaussian due to the central limit theorem.

3. Stochasticity of photoemission(2)

Continuous fluid

Photoemission process

3. Stochasticity of photoemission(3)

A relativistic electron accelerated in a magnetic field will radiate electromagnetic energy at a rate which is proportional to the square of the accelerating force.

<Averaged radiation power>

classical electron radius

$$P = \frac{2}{3}\frac{r_e c}{(m_0 c^2)^3} E^2 F_\perp^{\ 2} = \frac{2}{3}\frac{r_e c}{(m_0 c^2)^3}\frac{E^4}{\rho^2}.$$

<Averaged radiation energy per turn>

Averaged properties are smooth !

$$U = \int^T P dt = \int^L P \frac{dl}{c},$$

$$U(\mathrm{keV}) = \frac{88.5 E^4(\mathrm{GeV})}{\rho(\mathrm{m})}$$ for the case with the constant ρ.

3. Stochasticity of photoemission(4)

As a first approximation, we can assume that a photo-emission process is continuous. However, all electromagnetic radiation occurs in quanta of discrete energy. This quantization of the energy loss is characterized by instantaneous emission, random emission timing, and random quantized energy, etc.

<Mean square of energy loss fluctuation per unit time>

$$\langle Nu^2 \rangle = \frac{55}{24\sqrt{3}} \frac{U}{T} u_c \,, \qquad u_c = \frac{3}{2} \frac{\hbar c}{\rho} \gamma^3 \,.$$

u_c: critical photon energy ~ 3.2<u> <N>(phs/s) ~ 3.2P/ u_c

3. Stochasticity of photoemission(5)

u_c represents the photon energy at the peak brilliance of the BM radiation.

3. Stochasticity of photoemission(6)

Let's estimate the parameters!

<Case-1 E=1 GeV, ρ=5m>

$$U(\mathrm{keV}) = \frac{88.5\times 1^4}{5} = 17.7, \quad P(\mathrm{keV/s}) = \frac{U}{T=2\pi\times 5/c} = 1.7\times 10^8$$

$$u_c\,(\mathrm{keV}) = \frac{3}{2}\,\frac{6.85\times 10^{-19}\times 2.998\times 10^8\times 1957^3}{5} = \mathbf{0.44}$$

$$\langle Nu^2\rangle\,(\mathrm{keV}^2/\mathrm{s}) = \frac{55}{24\sqrt{3}}\,1.7\times 10^8 \times 0.44 = 0.99\times 10^8$$

$$\langle N\rangle(\mathrm{photons/s}) \sim 3.2\times 1.7\times 10^8 / 0.44 = \mathbf{1.2\times 10^9}$$

3. Stochasticity of photoemission(7)

SPring-8 parameter is the next example.

<Case-2 E=8 GeV, ρ=40m>

$$U(\mathrm{keV}) = \frac{88.5\times 8^4}{40} = 9.1\times 10^3,\quad P(\mathrm{keV/s}) = \frac{U}{T=2\pi\times 40/c} = 1.1\times 10^{10}$$

$$u_c\,(\mathrm{keV}) = \frac{3}{2}\,\frac{6.85\times 10^{-19}\times 2.998\times 10^8\times 15656^3}{40} = 29.6$$

$$\langle Nu^2\rangle\,(\mathrm{keV}^2/\mathrm{s}) = \frac{55}{24\sqrt{3}}\,1.1\times 10^{10}\times 29.6 = 4.31\times 10^{11}$$

$$\langle N\rangle(\mathrm{photons/s}) \sim 3.2\times 1.1\times 10^{10} / 29.6 = 1.2\times 10^9$$

4. Distribution of circulating electron beam(1)

6D-phase space volume of a single electron, ε_{s6} comprises of canonical variables (x, px(x'), y, py(y'), t, ps(ΔE/E)).
In an ideal case, 6D-phase space volume can be written by the product of areas of the three orthogonal 2D spaces ε_{sz}.

z=x,y,s

$$\varepsilon_{s6}=\varepsilon_{sx}\varepsilon_{sy}\varepsilon_{ss}$$

These are invariants of the motion for a conservative system.

4. Distribution of circulating electron beam(2)

You can understand the relation between the 2D phase space and the emittance by using a simple harmonic oscillator.

$$x = A \cdot \cos(\omega t + \phi_0)$$

$$\frac{dx}{ds} = x' = -A \cdot \omega \sin(\omega t + \phi_0)$$

$$\longrightarrow \quad \frac{1}{\omega}\frac{dx}{ds} = -A \cdot \sin(\omega t + \phi_0)$$

Action=A, Angle=$\phi = \omega t + \phi_0$,

$$\text{Invariant} = x^2 + (x'/\omega)^2 = A^2 = \frac{\text{circle area}}{\pi} = \varepsilon_{sx}$$

Distribution of circulating electron beam(3)

We need potential wells to stabilize three orthogonal oscillation modes to keep electron beam in a storage ring.

Quadrupole magnets generate the adequate potential wells for two transversal oscillation modes, which are called betatron oscillations in the horizontal and vertical planes.

RF acceleration electric field generates the adequate potential well for longitudinal oscillation mode, which is called a synchrotron oscillation.

Distribution of circulating electron beam(4)

SR light properties reflects the 3×2D phase space distribution of circulating electrons.

We use the following **three ensemble-averaged emittances** to express beam distribution in three orthogonal phase spaces.

$$\langle \varepsilon_{s6} \rangle = \langle \varepsilon_{sx} \rangle \langle \varepsilon_{sy} \rangle \langle \varepsilon_{ss} \rangle = \varepsilon_x \varepsilon_y \varepsilon_s,$$

ε_x : horizontal emittance (m rad)
ε_y : vertical emittance (m rad)
ε_s : longitudinal emittance (m rad)

4. Distribution of circulating electron beam(5)

Width of the Gaussian distribution of N circulating electrons is determined by the **dynamical equilibrium between the radiation excitation and damping**.

4. Distribution of circulating electron beam(6)

Remember the invariant of a harmonic oscillator.

$$\frac{d\langle A^2\rangle}{dt} = \frac{d\varepsilon_z}{dt} \qquad z = x,y,s.$$

Equilibrium condition:

$$\frac{d\langle A^2\rangle}{dt} = \lim_{\Delta t\to 0}\frac{\langle (A+\Delta A)^2 - A^2\rangle}{\Delta t} = \lim_{\Delta t\to 0} 2\frac{\langle A\Delta A\rangle}{\Delta t} + \lim_{\Delta t\to 0}\frac{\langle \Delta A^2\rangle}{\Delta t} = 0$$

Damping term

Averaged energy dissipation

Excitation term

Quantum effect

4. Distribution of circulating electron beam(7)

Energy spread $\sigma_{\Delta E}$

$$\lim_{\Delta t \to 0} 2\frac{\langle A\Delta A\rangle}{\Delta t} = -2\frac{\langle A^2\rangle}{\tau_\varepsilon}, \quad \tau_\varepsilon = \frac{E\times T}{U},$$

$$\lim_{\Delta t \to 0} \frac{\langle \Delta A^2\rangle}{\Delta t} = \langle Nu^2\rangle = \frac{55}{24\sqrt{3}}\frac{U}{T}u_c.$$

Since σ_ε is not the emittance, phase average factor should be considered, $\sigma_{\Delta E}^2 = \frac{\langle A^2\rangle}{2}$,

$$\sigma_{\Delta E} = \sqrt{\frac{55}{96\sqrt{3}}E\times u_c}, \quad \sigma_{\Delta E/E} = \sqrt{\frac{55}{96\sqrt{3}}\frac{u_c}{E}}.$$

4. Distribution of circulating electron beam(8)

Bunch length σ_τ

σ_τ has two components;

$\sigma_{\tau 1}$= time spread by energy spread

$\sigma_{\tau 2}$= time spread by stochastic photoemission

$\sigma_{\tau 1} >> \sigma_{\tau 2}$

$$\sigma_\tau = \frac{\alpha}{\omega}\sigma_{\Delta E/E}$$

10^{-4} 10^{-3} 10^{4}

= ~10 psec

@ SPring-8

Phase space of synchrotron (energy) oscillation

4. Distribution of circulating electron beam(9)

Horizontal emittance ε_x

Excitation: due to the discrete energy jump + energy dispersion

Damping: due to the decrease of transverse momentum by the photoemission + acceleration along the running direction

Excitation Damping

4. Distribution of circulating electron beam(10)

For the typical magnetic lattice structure (Chasman Green: CG) based storage ring, the horizontal minimum emittance is written by

$$\varepsilon_{x\,min}\ @\text{general} \sim \frac{1}{2}\varepsilon_{x\,min}\ @\text{achromat}$$

$$= \frac{C_q \gamma^2}{8\sqrt{15}\ J_x}\theta_b{}^3$$

C_q: Quantumn constant 3.832×10^{-13} (m)
θ_b: Deflection angle of a single bending magnet (rad)
Jx: Horizontal damping partition number ~ 1

4. Distribution of circulating electron beam(11)

Chasman-Green (CG) lattice is the most popular magnet cell structure for a low emittance SR souce, where a achromatic arc is composed of a pair of bending magnets.

2 bends: CG or CBA
3 bends: TBA
4 bends: QBA

$\varepsilon_{x\,min} \propto \theta_b^{\,3}$

θ_b smaller with same cell No.

4. Distribution of circulating electron beam(12)

Vertical emittance ε_y

ε_y has two components;

ε_{y1}= vertical emittance by stochastic photoemission

ε_{y2}= vertical emittance by HV coupling

Usually, $\varepsilon_{y1} << \varepsilon_{y2}$

The Angular divergence
In the vertical plane is ~ $1/\gamma$

1-GeV storage ring
γ=1000/0.511~2000

$1/\gamma \sim 5\times10^{-4}$

Principally, ε_y =1/1000 ε_x is possible

4. Distribution of circulating electron beam(13)

Vertical emittance is determined by magnetic error components mixing the horizontal and vertical betatron oscillation. The main sources are vertical misalignments of sextupole magnets and rotational errors of quadrupole magnets.

The effect of these error fields can be corrected to **0.1 % level** by the combination of beam response analysis and skew quadrupole corrector magnets.

One-dimensional diffraction limited X-ray beam is now available

5. Current limiting factors(1)

$B \propto I_b$ What limits the stored current ?

(1) The circulating electrons trapped in the RF acceleration potential forms the bunch.

(2) The electron-electron scattering scarcely causes the emittance growth and instability in high energy region.

(3) The electron bunch circulates at the inside of the vacuum chamber, which has lots of non-smooth structures such as bellows, valves, cavities.

(4) The indirect interaction of the electrons via the external boundary determines the current upper limit.

5. Current limiting factors(2)

Wakefield by the circulating electron beam

Example: accelerating structure

The electron bunch leaves a part of beam energy as a electro-magnetic (wakefield) field which perturbs the successive bunches.

5. Current limiting factors(3)

Average current limit by coupled bunch instability

(1) The circulating electron bunches have a wide frequency range from revolution to higher frequencies up to 10 GHz corresponding to the bunch length.

(2) These bunches excite harmful wakefields in high-Q structure or at narrow gap ID with a quite large impedance at lower frequencies.

(3) The excited wakefield causes the coherent instability.

5. Current limiting factors(4)

Image of the coupled bunch instability

High Q structure (RF cavity)

Narrow gap undulator

5. Current limiting factors(5)

Peak current limit by single bunch instability

(1) The wakefield excited in non-smooth structures, of which size is much smaller than the bunch length, works on the bunch as an inductive force.

(2) This wakefield generates the energy chirp in the bunch and deforms the bunch distribution.

5. Current limiting factors(6)

Towards the high stored current

Passive

(1) Suppression of harmful impedance

(2) Landau damping enhancement

Active

(3) Introduction of some feedback system

At SPring-8,
average current ~120 mA (target 200 mA)
Single bunch current ~5mA (Max. 15mA)

High current vs low emittance, which is better ?

6. Beam stability(1)

Beam stability is quite important for effective use of sharp (brilliant) SR

Perturbation sources

- **Compressors & Pumps**
- **Magnet current ripple**
- **Vibration by cars, trains, ..**
- **Temperature drift**
- **Earth tide**
- **Earthquake**
- **etc…**

Sometimes, river also becomes a serious perturbation source….

6. Beam stability(2)

To supply stable brilliant SR, we have continued to improve the beam stability from multi-angles.

6. Beam stability(2)

Beam energy stabilization by RF frequency FB

6. Beam stability(3)

Beam orbit stabilization by the global COD FB

6. Beam stability(4)

Beam orbit stabilization by the source suppression

6. Beam stability(5)

Making IDs transparent for users

6. Beam stability(6)

Introducing top-up operation

6. Beam stability(7)

Beam stability of a few μm was achieved

7. New idea towards higher beam performances(1)

- EUV-FEL and XFEL
- 4-dimensional phase space manipulation for an intense micro-beam

Towards XFEL

(a) Oscillator

(b) Single Pass (SASE) FEL

(a) Mirror development for EUV~X-ray region
(b) High quality electron beam for a high FEL gain and long undulators with high field quality

Laser amplification by SASE

4-dimensional phase space manipulation

Motivation

In the SPring-8 storage ring, spatial coherence of X-ray SR is limited by horizontal electron beam emittance, being much larger than photon beam emittance. On the other hand, vertical one is smaller compared with photon beam emittance. The spatial coherence is potentially improved by adjusting the miss-match between two kinds of emittance, *i.e.*, optimizing the electron distribution in the 4-dimensional transverse phase space.

Spatially coherent X-ray from unmatched electron beam

Spatially coherent X-ray from matched electron beam

How to realize

Magnet-free long straight section of 30m + 4-dimensional phase space transformer